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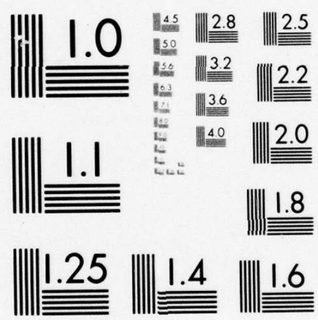

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RESEARCH IN GRAVITY WAVES AND AIRGLOW PHENOMENA

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23 November 1976

Final Report
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(1) An analysis of the behavior of 6300 Å OI and 5200 Å NI emission lines under the influence of Atmospheric Gravity Waves.

(2) The propagation of gravity waves and the fully guided, partially guided as well as free modes are treated by a "potential" model.

(OVER)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research in this final report may be divided into three parts. The first part deals with the behavior of airglow (mid-latitude) under the influence of atmospheric gravity wave model is assumed. Our principal objective here is to examine its effect on the ionosphere and the 6300 Å OI 5200 Å NI emission lines. The results were found to be in good agreement with Dach's (1968) experimental observations.

The second part deals with a new approach for treating gravity waves. The

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(3) The effect of atmospheric gravity waves on the natural oscillation of the atmosphere.

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gravity wave problem can be reduced into a multi-channel scattering problem with effective potentials for each channel. The "potentials" for the gravity channel corresponding to different exospheric temperatures were obtained from the 1972 Cospar models. The results show (1) above periods of 1/2 an hour the "potentials" no longer change in shape. (2) There are no strictly fully ducted modes. The S_1 and S_2 modes exist only as partially ducted modes. The horizontal phase velocities, however, are identical with those found by Francis (1973b). (3) There is no ducted mode for the F_2-region.

The third part is an investigation of the non-linear effects of gravity wave on natural oscillations. Airglow (mostly 5577 Å OI) oscillations often show a small period (8-14 min.) oscillation superimposed on a long period ($\sim$ hour) oscillation. The short period oscillation is sometimes "excited" by the long period oscillation as the latter increases in amplitude. We have approx. found that for Richardson's number much greater than 1/2 when instability does not occur "self excitation" of natural oscillations can be induced by gravity waves with sufficiently large amplitude.

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FINAL REPORT

Summary and Analysis

The research reported here may be divided into three sections. The first deals with the effect of atmospheric gravity wave on airglow. The work results in a paper entitled "On The Behaviour of Airglow Under The Influence of Gravity Waves". (Porter, Silverman and Tuan (1974)). The second section deals with "potential" model for gravity waves (Yu, Tai and Tuan (1975)). Most of the work for this is finished. Upon completion we intend to write a long paper on this new approach to gravity wave problems. The third section deals with the non-linear effects of gravity waves, in particular their effects on natural atmospheric oscillations and airglow, (Tuan, Hedinger, Tai and Silverman (1975)). A preprint is enclosed with this Final Report. We intend to continue working on this problem which shows promise for handling a much more general variety of non-linear gravity wave problems.

I. On the Behaviour of Airglow under the Influence of Atmospheric Gravity Waves

The development of the technique for treating the effect of gravity waves on ionosphere has been given in some detail in the final report in (1973), (see also Porter and Tuan (1974)). We have since applied the Green's function technique developed in that paper to the 6300 Å OI and the 5200 Å NI emissions in mid-latitudes.

Our procedure is based on the assumption that these two highly forbidden emission lines can only occur in the F-region and the upper F-region where our theoretical calculations for the behaviour of electron density profiles under the influence of a fully ducted long period gravity wave (Porter and Tuan (1974)) is most valid. For long period gravity waves the continuity equation for oxygen concentration in the ^1D states reduces to the quasi-equilibrium equations of Petersen, Van Zandt and Norton (1966). Together with the time-dependent electron density profiles

the emission rate for 6300 Å OI may be calculated. A rather stringent test of our theoretical calculations has been provided by the experimental observation of Dachs (1968) who has obtained not only the 6300 Å observations as a function of time but also the simultaneous f_oF2 and h'F2 variations with time. With a suitable choice of only two parameters, (the scale height H = 65 Km, the scale height factor p = 1.5), the theoretical results for simultaneous variations in 6300 Å OI, f_oF2 and h'F2 appear to agree very well with the corresponding experimental observations. Considering how few free parameters are used in comparison with the diversity of the experimental observations (both airglow and ionospheric parameters), the agreement must give us confidence in our basic theory.

For the 5200 Å NI we can no longer use the quasi-equilibrium theories of Peterson, Van Zandt and Norton (1966), since the half life of the $^2D_{5/2}$ state is very much larger than the period of the gravity-wave oscillation. This is especially so at high altitudes where quenching can no longer compensate for the very slow radiative loss. By making use of the theories of Hernandez and Turtle (1969) which neglects the diffusion terms in the continuity equation for atomic nitrogen concentration in the ^2D state but includes the time dependence, we can calculate the ^2D nitrogen state concentration. The effect of electrons on the nitrogen concentration is to quench the optically forbidden state through superelastic collisions. Thus, a sinusoidally varying electron density concentration produced by the gravity waves would also produce a sinusoidally varying 5200 Å nitrogen emission line.

Several interesting results have come out of this calculation. (1) The importance of electron quenching for this highly metastable nitrogen state has effectively kept the peak of the 5200 Å luminosity profile relatively low even though the very long half life may at first seem to imply a high luminosity peak. In fact the calculated peak is comparable to the 6300 Å OI luminosity peak. This is consistent with the observations of Weill and Christophe-Glaume (1967) who argued from geometric considerations that the 5200 Å peak should only be about 12 km above the

6300 Å peak if the speed of the travelling ionospheric disturbance is assumed to be the same for both emissions. (2) Because of the effect of quenching and the lack of O_2^+ and NO^+ molecules at high altitudes to produce airglow, the vertical movements of the luminosity profiles is considerably more restricted than the movements of the peak electron density profile. (3) The rate of change of total columnar electron content is always negative in the absence of incident electron flux. This means that in the presence of a gravity wave, the oscillation of the columnar electron content decreases monotonically with height.

The result is significant in that any non-monotonic oscillations in the total electron content cannot be caused by gravity waves. Thus, a measurement of the columnar electron content variation with time can determine whether a gravity wave or some incident electron flux is causing the oscillations. (4) There is always a phase advance in the airglow fluctuations with respect to the change in the total electron content. The reason for this is discussed in detail in the enclosed preprint.

II. On "Potential Model" for Gravity Waves

Introduction

Most theoretical studies on the propagation of gravity waves through the atmosphere are classified into the following categories; the ray theory, the WKB method and the multi-layer model. Of these methods the multi-layer full wave approach (Francis, S.H., (1973), Klostermeyer, J., (1972a,b) Volland, M. (1969), Midgeley, J.E. and Liemohn, H.B. (1966), Friedman, J.P. (1966)) is perhaps the most complete in that the formalism allows the waves to be scattered into different channels. For instance, a gravity wave may be scattered or transmitted as a heat conduction wave and vice versa. The basic assumption used for these full wave model calculations is that the atmosphere is horizontally stratified and may be divided into thin isothermal layers in which we may assume the various atmospheric parameters to be constant. The waves are then joined together at the

interface by appropriate boundary conditions. There are a number of difficulties associated with such an otherwise excellent approach.

(1) The sharp step function change in the parameters such as temperature etc. across different layers would produce δ-functions in their height derivatives (Hines (1973)). Although justification for some special use of the multi-layer model have been given by a number of authors (e.g. Pierce, (1966)), it may be desirable to verify these justifications by a totally different approach.

(2) There has been some uncertainty about the boundary conditions to be used at the interface between adjacent layers, owing to the perturbation of the wave on the ambient atmosphere, (Francis, 1973, Volland, 1969, Friedman, 1966).

(3) A good deal of uncertainty seems to exist for the so-called upper boundary conditions. The spectrum for the guided modes is dependent on whether this upper boundary condition is assumed to be a free surface boundary, a rigid surface boundary or the radiation condition, (Friedman (1966), Tolstoy and Pan (1970)).

Basically, there is very little one can do about the upper boundary condition as long as one sticks to the usual formalism of multi-layer method as applied to the system of coupled first order differential equations. The reason is that the solution is invariably a plane progressive wave and in a numerical solution one must follow such a wave up the atmosphere until an "upper" boundary is reached. In reducing the problem to a second order Sturm-Liouville equation we may obtain "standing wave" solutions for the discrete eigenstates. Outside the range of the "potential" we may just simply use the "outgoing" boundary conditions commonly used in atomic and nuclear physics.

(4) Different nomenclature has been used for waves considered not to have the same guiding mechanism. A surface wave, for instance, is a wave which is vertically evanescent and is considered to be guided by say a temperature discontinuity, (e.g. the sharp temperature change at the base of the thermosphere).

Such a ducting mechanism has been originally suggested by Thome (1968) to account for large scale TID's in the F-region. The mechanism has been developed later by Francis for a more realistic temperature profile (Francis 1973). Such a surface wave is not considered to have the same guiding mechanism as for instance, the S_2 mode at lower altitude. This mode is supposed to be guided by two boundaries instead of one, the mesopause and the ground which form the upper and lower boundaries of a wave guide. Our present approach should settle the problem whether these apparently different waves with different names are in fact guided by fundamentally different mechanisms.

(5) Some considerable controversy exists for the ducting mechanisms for medium scale TID's. Friedman (1966) maintained that medium-scale TID's are caused by strongly ducted gravity waves which have rather low horizontal phase velocity (less than 250 m/sec). Francis (1973) has maintained that gravity waves with so low a horizontal phase velocity cannot be ducted in the usual way. It is in fact refracted around the earth by the changing direction of the earth's radius of curvature.

(6) So far as we know there are no direct measurements of the presence of gravity waves at F-region heights. Usually, the waves are detected indirectly through its effect on the electron density concentration which can then be measured by means such as the incoherent backscatter. A knowledge of the phase front of such waves can help us determine the degree of ducting. In general, it is not easy to deduce the direction of the phase front from the phase of the TID. Porter and Tuan (1974) have shown that if a purely vertical phase front is assumed for the gravity wave, the TID would have a nearly horizontal phase front in the F-region peak. In fact phase variation with height is almost identical to Fig. 14 in Francis' paper (Francis 1973) (see also Figure 7, Porter and Tuan 1974). Thus, just because the phase front of TID may be nearly horizontal at the F-region peak,

does not imply that the gravity wave has a horinzontal phase front. Thome (1968) has used a step function temperature profile to obtain a single fully ducted mode. Francis (1973) has used the 1965 Cospar Model and obtained three partially ducted modes (for long period gravity waves).

In view of all these difficulties some of which apparently arise from the multi-layer model itself, we will investigate the behaviour of atmospheric gravity waves by a totally different approach. We will treat the gravity wave propagation through the atmosphere as a multi-channel scattering problem with effective "potentials" for each channel. For this report we will restrict ourselves to altitudes below 500 Km where we may neglect viscosity. The gravity waves may then be scattered into the heat conduction wave channel and vice versa. The problem becomes very similar to scattering in atomic or nuclear physics. In first approximation we will decouple the heat conduction wave channel from the gravity wave channel by assuming a lossless atmosphere. For the gravity wave channel, we obtained the "potentials" for Summer and Winter Seasons as well as different exospheric temperature from the 1972 Cospar Models. The problem may then be treated as a scattering process (free modes), a resonance process (partially guided modes) or a bound-state problem (fully ducted modes).

We will investigate the behaviour of this potential for different gravity wave frequencies as well as for different seasons and different exospheric temperatures. We will also determine the number of bound (fully guided) and resonance (partially guided) states together with the behaviour of the pressure variation as a function of height for those states. We will also show that for long period waves the boundary conditions together with the potential imply the existence of a lowest ground state belonging to the lowest eigenvalue. Since the eigenvalue is in units of inverse horizontal phase velocity, there exists an upper limit to the horizontal phase velocity for guided modes. The free modes also have an upper limit (somewhat higher) which depends on the exospheric temperature. We will also develop an

approximate semi-empirical dispersion relation for a lossless atmosphere for altitudes higher than 130 Km where the "potential" becomes very flat and structureless. With this dispersion relation we hope to show that an F-region gravity wave can to some extent be ducted by the curvature of the earth and that, in fact, the sharp temperature change at the base of the thermosphere as given by the Cospar models cannot produce a sufficiently deep "Potential well" at the F-region to guide a gravity wave.

We will consider just how sharp the temperature change will have to be in order to produce a well deep enough to guide the F-region gravity wave.

Theory

As a first step, we shall consider only two channels, the gravity wave and the heat conduction channel. This is not a bad approximation (Volland, (1969)) for altitudes below 500 Km. It can be shown that for horizontally stratified atmosphere the coupled hydrodynamic equations can be reduced to the following coupled second order eigenvalue equations:

$$\frac{\partial^2(\Delta p)}{\partial z^2} + \alpha \frac{\partial(\Delta p)}{\partial z} + \beta(\Delta p) - K_x^2 (\Delta p) = A(\Delta T) + B \frac{\partial(\Delta T)}{\partial z} \tag{1}$$

$$\frac{\partial}{\partial z}\left[\kappa \frac{\partial(\Delta T)}{\partial z}\right] + D(\Delta T) - \kappa K_x^2 (\Delta T) = \gamma(\Delta p) + \delta \frac{\partial(\Delta p)}{\partial z}$$

where $\alpha,\beta,\gamma,\delta,A,B,D$ are known functions of temperature, mean molecular mass, ratio of specific heats and thermal conductivity. K_x is the horizontal wave number and κ is the thermal conductivity. Equation (1) contains both the gravity wave solution and the heat conduction wave solution. Each type of wave would have two linearly independent solutions corresponding to upward and downward propagation. In the case when $\kappa = 0$ we no longer have heat conduction. ΔT may then be immediately eliminated and we would obtain a single second order eigenvalue equation whose solutions are just the gravity waves.

Since the boundary conditions for the dependent field variables ($\Delta p, \Delta T$, etc.) are just the continuity of the field variable together with their hydrodynamic derivatives, (Thome, (1968)), they are essentially the same as the quantum mechanical boundary condition. Thus, the eigenvalues k_x^2 which are just the horizontal wave numbers may form a discrete set corresponding to "ducted" modes or a continuum corresponding to "free" modes.

(1) Discrete (bound or resonance) States

When the thermal conductivity κ is set equal to zero, equation (1) will immediately reduce to the following equation

$$\left[\frac{d}{dz} Q(z) \frac{d}{dz} - v(z) + \lambda^2\right]\psi = 0 \tag{2}$$

where $\psi = \omega\sqrt{\left(\frac{f}{\rho_o}\right)} \exp[-\frac{1}{2}\int f'/f\, dz]\Delta p$

$f = \frac{\gamma}{C}(\omega_b^2 - \omega^2)$

ω_b = Brunt-Vaisala frequency

$Q = \frac{1}{\omega_b^2 - \omega^2}$

$\lambda_x^2 = \left(\frac{K_x}{\omega}\right)^2 = \frac{1}{(v_{phx})^2}$

$$v(z,\omega) = \frac{1}{\omega_b^2 - \omega^2}\left\{\left(\frac{c^{2''}}{c^2}\right)\left[\frac{1}{2} - \frac{(\gamma-2)g^2}{2c^2}\frac{1}{\omega_b^2-\omega^2} - \frac{1}{2}\frac{c^{2'}}{c^2}\frac{g}{\omega_b^2-\omega^2}\right]\right.$$
$$+\left(\frac{c^{2'}}{c^2}\right)\left[\frac{(\gamma-2)g\omega_b^2}{2c^2(\omega_b^2-\omega^2)} + \frac{1}{2}\left(\frac{c^{2''}}{c^2}\right)\frac{\omega_b^2}{\omega_b^2-\omega^2} - \frac{1}{4}\left(\frac{c^2}{c^2}\right)\right]$$
$$\left. + \frac{(\gamma-2)^2g^2}{4c^4} - \frac{\omega_b^2-\omega^2}{c^2}\right\} \tag{3}$$

Immediately from equation(2), we learn that (1) for ducted or partially ducted modes, since ψ obeys the usual boundary conditions, the eigenvalue $\lambda_x^2 = \frac{1}{(v_{phx})^2}$ will be discrete which means that the horizontal phase velocity will also be discrete.

Furthermore, there is clearly a "ground" state corresponding to maximum horizontal phase velocity.

(2) For long period gravity waves, $\omega_b^2 >> \omega^2$, the "potential" becomes independent of the frequency of the gravity wave. In practice if the period of the gravity wave is greater than 30 minutes, the "potential" is, for all practical purposes, independent of ω. This means that for such waves there is no longer significant dispersion. It is easy to show that for an isothermal atmosphere, and constant sound speed profile equation (2) would reduce to the Hines' dispersion relation if we substitute in a plane wave solution. The profiles of temperature, mean molecular weight, ratio of specific heats and gravitational constant are taken from the 1972 Cospar models, and the U.S. stand and atmosphere (1965). Fig. 1 shows the temperature profiles for the Summer Season with exospheric temperature ranging from 600°K to 2200°K. Fig. 2, 3 and 4 show the "potentials" for long period gravity waves (> 30 min.) when they are no longer dependent on gravity wave frequency. They correspond to exospheric temperatures of 600°K, 1800°K and 2200°K respectively. The dashed curve is the Summer "potential" while the solid line is the Winter "potential".

These potential curves above can already provide us with a lot of information. One at once sees that there are no strictly fully ducted modes since clearly only "resonance" states can exist for such potentials. For long period gravity waves, there is clearly an upper limit to the horizontal phase velocity as well as an upper limit to the horizontal phase velocity of the guided modes. We should emphasize here that in neglecting viscosity and heat conduction, our "potential" models are good approximations only below 150 Km. Above 150 Km, the dissipative losses become important. It is possible to show that such losses form an imaginary part in addition to our real "potential" given by equation (3). We may then in general use the optical model treatment for the entire problem.

We will for the moment consider only the real "potential" given by equation (3). If we use a Winter temperature profile with an exospheric temperature of $1500^{o}K$, the potential for a gravity wave with a two-hour period is given by Fig. 5. The discontinuity at the ground surface provides a δ-function "potential" which may be replaced by the boundary condition,

$$\left.\frac{\partial\psi}{\partial z}\right|_{z=o} = \left\{\frac{1}{2}\left(\frac{\gamma g}{c^2}\right) + \frac{c'}{c} - \frac{g}{c^2}\right\} \psi \Big|_{z=o} \tag{4}$$

The important feature of our "potential" is that it becomes quite flat and constant above 200 Km. The solution above this height can be approximated very well by plane waves. The problem of determining ducted or partially ducted modes thus reduces to determining the bound or resonance states for atomic and nuclear physics. We may hence use phase shift analysis to determine the resonance states. The phase shift may be computed from the usual equation

$$K_z \cot\left(K_z R + \delta(\lambda^2)\right) = \left.\frac{\psi'}{\psi}\right|_R \tag{5}$$

where R is any altitude in the region where the potential is effectively flat. A resonance occurs whenever δ goes through $\frac{\pi}{2}$. Fig. 6 shows a plot of the phase shift computed from equation (5). The curve shows that the phase goes through a very sharp change of π radians at $\lambda^2 = 10.43$ and $\lambda^2 = 14.33$ corresponding to horizontal phase velocities of 309.7 and 264.2 meters per second respectively. This agrees very closely with the S_1 and S_2 modes of Francis (1973). A rather faint suggestion of a resonance occurs for $\lambda^2 \sim 26$ but the resonance is too broad to be considered as anything much more than background. Something similar has been discussed by Francis with the multi-layer method. The very sharp resonance at $\lambda^2 = 10.43$ is the well known Lamb mode. As can be seen from the location of the two resonances in Fig. 5 the Lamb mode is not really fully ducted. In fact, there are no fully ducted modes as already mentioned not even the Lamb mode which has the lowest eigenvalue. Considering the fact that we have not used similar temperature profiles as Francis

and that we are only considering the real part of the potential the agreement would be another confirmation that the imaginary part of the potential (the dissipation) is relatively unimportant below 150 Km. We will hence identify our resonance as the S_1 and S_2 modes. The eigenfunction ψ_1 for the S_1 mode is given in Fig. 7 which shows the very high amplitude near the ground level. Above 150 Km when the potential is very flat with no structure whatsoever, the amplitude of the eigenfunction becomes exceedingly small. However, there is clearly a small finite possibility for leakage and the eigenfunction is definitely that of a resonance state. Fig. 8 shows a plot of the S_2 mode eigenfunction ψ_2. Once again, the amplitude becomes smaller above 150 Km and is quite large below that height level. However, there is clearly considerably more leakage. ψ_2 has large amplitudes at about 50 Km where there is a deep well (Fig. 5) formed by the hump at the stratopause. However the most important guiding mechanism for this wave still comes from the δ-function potential at the ground level. For this reason, the partially guided gravity waves for both the S_1 and S_2 modes should be readily observable at the ground level.

(4) <u>The Guided Mode at the F-Region</u>.

At F-region altitudes dissipative losses due to heat conduction, viscosity and ions drag become very important. Perhaps the most important of these losses is heat conduction (Volland (1969), Francis (1973a)).

Since at the moment we are only concerned with the real part of the potential, we will consider the effect of the temperature structure as well as the variation in the mean molecular mass on long period gravity waves with high horizontal phase velocity.

There is a general belief that the sharp temperature drop at the base of the thermosphere can be considered as some form of discontinuity which can guide a surface wave. Thome (1969) has used a step function temperature profile to obtain a single fully guided mode. Later Francis (1973) by using the 1965 Cospar model

together with assumptions on losses due to heat conduction and viscosity succeeded in obtaining three partially ducted modes. Both Thome and Francis used the multi-layer method, although in the case of Thome there were only two layers, since a step function temperature profile was used.

Within the physical framework of a potential model, a step function temperature profile is just a δ-function potential. It is well known that a δ-function potential can have a single bound state with finite eigenvalue. Thus, the single mode of Thome is not surprising. Furthermore, a well behaved solution for a δ-function potential must consist of decaying exponentials which implies that the wave numbers are purely imaginary.

Since this is just the definition of a surface wave ducted by a single discontinuity, we see that within the present physical framework, there is no need to consider a surface wave as something separate from other guided waves.

A glance at Fig. 2, 3 and 4 immediately reveals the fact that the F-region well if it exists is very shallow indeed. Part of the reason why it is so shallow is that the "potential" as given by equation (3) really depends on derivatives of the speed of sound c rather then the temperature profile. The variation of the mean molecular mass with altitude is such that the sound speed profile has a much more gradual slope than the temperature profile.

However, for high exospheric temperatures, it is possible to show that an exceptionally shallow but broad well does exist at the F-region which corresponds to the temperature rise above the mesopause. Such a potential well can be very accurately approximated by the Morse Potential given by:

$$V = V_o + a[e^{-2\alpha(z-z_o)} - 2e^{-\alpha(z-z_o)}] \qquad (\ )$$

where z_o = height at which the minimum of the well occurs

V_o = some scaling constant.

The quantities a and α are the two Morse potential parameters. One can then show that the criteria for the existence of bound states in terms of the Brunt frequency ω_b is given by:

$$N = \frac{\sqrt{a(\omega_b^2 - \omega^2)}}{\alpha} > \frac{1}{2}$$

where ω is the frequency of the gravity wave. Table I shows the values for N for long period (> $\frac{1}{2}$ hour) gravity waves and exospheric temperatures of 2000°K and 2200°K respectively.

TABLE I

Exospheric Temp.	2000°	2200°
Z_o	272 Km	278 Km
α	1/70 Km	1/74 Km^{-1}
A	$7.396 \times 10^{-2} Km^{-2} sec^{-2}$	$8.39 \times 10^{-2} Km^{-2} sec^{-2}$
V_o	$6.86 \times 10^{-1} Km^{-2} sec^{2}$	$5.957 \times 10^{-1} Km^{-2} sec^{2}$
N	0.133	0.146

It is clear that both values of N are less than 0.5 the minimum required for a bound state. For lower exospheric temperatures N becomes smaller still.

In order to determine just how sharp the temperature drop will have to be at the base of the thermosphere in order to produce a well sufficiently deep to guide a gravity wave we used an analytic temperature profile given by Fig. 9a. Fig. 9b shows the corresponding well with a minimum at about 140 Km. It is clear that such a well can only produce fully ducted modes with horizontal phase velocities greater than 1000 meters/sec ($\lambda^2 < 1$). We are not aware of observations which support this.

To explain Thome's (1969) observations which gave pretty convincing evidence that at least a partially ducted wave at the F-region altitudes exist with horizontal

phase velocities of the order of 700 m/sec and a period of about 2 hours, we must point out that the real part of the "potential" only becomes predominant below 150 Km. Above 200 Km the imaginary part of the potential becomes much more important than the real part. If we confine our attention for the time being to below 150 Km, we see immediately from the potential that, for $\lambda^2 < 8$ which corresponds to $v_{phx} > 350$ m/sec, there is little gravity wave below 120 Km. Fig. 10 shows a diagram of a gravity wave with a 2 hour period and a horizontal phase velocity of 707 m/sec. which is rather similar to Thome's (1968) observed gravity waves. We see that there is relatively little wave amplitude below 130 Km. This is in good agreement with Thome's experimental observation of the gravity wave on May 13-14 and on June 9-10, 1964. The May 13-14 wave has horizontal phase velocity of 700 m/se and induces imperceptible oscillation at 134 Km where the electron density is still relatively significant. The June 9-10 wave has a horizontal phase velocity of 600 m/sec during the day and again induces no oscillation at 131 Km. From Fig. 10, we see that waves with such horizontal phase velocities do not have very much amplitude below 130 Km. In fact, the gravity wave becomes evanescent below 120 Km. These results are also in good qualitative agreement with the G_o', G_1', and G_2' pseudomodes of Francis (1973).

Above 150 Km we can no longer ignore the imaginary part of the potential. To a first approximation, we may use the WKB approximations which can immediately show that the imaginary part produces severe amplitude damping with height as well as a change of phase in the eigenfunction ψ. The eigenvalues λ^2 of the eigenstates will also change. However, since imaginary part W of our potential is so large at higher altitudes that we can no longer ignore second order terms in the scattering amplitude A. We may write

$$A = \langle\phi_b|v_t - v_t G_o v_t + \ldots.|\phi_a\rangle$$

where

$$v_t = v + iW,$$

and G_o is the Green's function obeying the same boundary condition as ψ at the origin and an outgoing boundary condition at $z \to \infty$. G_o satisfies the equation

$$\frac{d}{dz} Q(z) \frac{d}{dz} + \lambda^2 \quad G(z,z',\lambda^2) = -\delta(z-z') \quad .$$

The second order term contains four terms, one of which is the real term WG_oW which can be very important and may produce resonance states. Since $W \to 0$ as $\kappa \to 0$, we know that such resonance states corresponding to partially guided modes are strictly produced by dissipation.

In summary for the F-region long period gravity wave, we are in agreement with both the experimental observations of Thome as well as the theoretical treatment of Francis below 140 Km. Above this height level we are not in any real disagreement with Thome since we would have obtained a single fully ducted mode if we had used a step function temperature profile. However, we are in sharp disagreement with Francis (1973) who has obtained three partially ducted modes which he has labelled G_o', G_1', G_2' and has furthermore maintained that they are essentially produced by the temperature discontinuity at the base of the thermosphere. Since we have both used realistic temperatures as well as speed of sound profiles, it is difficult to see how such differences may be easily reconciled. As already mentioned, the only way we can obtain ducted modes is through the second order effect of the imaginary part of our potential. Since W, the imaginary part of the potential, is large, it is certainly possible that some kind of ducting corresponding to a resonance state may be feasible. However such resonance states are not caused by the temperature change at the base of the thermosphere.

(5) <u>The effect of the earth's curvature on F-region gravity waves</u>.

Although the temperature gradient appears to be insufficient for ducting longer period atmospheric gravity waves in the F-region, there are means other

than atmospheric structure which may be able to produce sufficient ducting to explain the observed behaviour. Practically all observed F-region gravity waves are ducted only about 1/4 of the way around the earth. The distance is of the order of 10,000 Km. Fortunately, for the F-region, the "potential" is very nearly flat and ω_b also becomes a very slowly varying function of height. Thus, we may to a first approximation assume that Q(z) and v(z,w) are both constant. From equation (2) we may immediately derive the following semi-empirical dispersion relation for the F-region

$$k_z^2 = k_x^2\left(\frac{\omega_b^2}{\omega^2} - 1\right) - (\omega_b^2 - \omega^2)V \tag{6}$$

where V is a constant and may be taken from the flat part of the "potential" curves. We may now adopt a similar procedure as Francis (1972). It is possible to show that the group velocities are given by:

$$v_{gx} = \left.\frac{\partial\omega}{\partial K_x}\right|_{K_z} = \frac{k_x}{k_x^2} - \frac{k_x\left(\frac{\omega_b^2}{\omega^2} - 1\right)}{\omega V - k_x^2\left(\frac{\omega_b^2}{\omega^3}\right)}$$

$$v_{gz} = \left.\frac{\partial\omega}{\partial K_z}\right|_{K_x} = \frac{k_z}{\omega V - k_x^2\left(\frac{\omega_b^2}{\omega^3}\right)} \tag{7}$$

If we introduce x as a variable defined by

$$x = R_E\theta$$

where R_E is the radius of earth and θ is the angle with respect to say the axis of rotation, and z as the vertical height normal to the earth's surface, then

$$\frac{dz}{dx} = \frac{v_{gz}}{v_{gx}} = -\frac{k_z}{k_x\left(\frac{\omega_b^2}{\omega^2} - 1\right)} \tag{8}$$

Since V is a constant in equation (6), we may immediately obtain the following equations for K_x and K_z

$$\frac{dK_z}{dx} = \frac{K_x}{R_E}$$

$$\frac{dK_x}{dx} = \frac{K_z}{R_E^2} \frac{\omega}{\omega_b - \omega} \tag{9}$$

The above equations are identical to those of Francis (1972).

Solving for K_x and K_z, we obtain

$$K_x = A\, e^{\left[\frac{\omega}{(\omega_b^2 - \omega^2)^{1/2}}\right] \frac{x}{R_E}} + D\, e^{-\left[\frac{\omega}{(\omega_b^2 - \omega^2)^{1/2}}\right] \frac{x}{R_E}} \tag{10}$$

$$K_z = \frac{\sqrt{\omega_b^2 - \omega^2}}{\omega} \left(A\, e^{\left[\frac{\omega}{(\omega_b^2 - \omega^2)^{1/2}}\right] \frac{x}{R_E}} - D\, e^{-\left[\frac{\omega}{(\omega_b^2 - \omega^2)^{1/2}}\right] \frac{x}{R_E}}\right)$$

Since D > A and the exponent is much less than one for $x \leq 10{,}000$ Km, K_z is negative. There is hence always a strong forward tilt as shown in Fig. 11. If we assume that the ray path begins at 160 Km, it will reach a height of only about 310 Km after travelling 10,000 Km i.e. 1/4 of the way around the earth.

The gravity wave here is assumed to have a two hour period and a horizontal phase velocity of 723 m/sec. Since these numbers are similar to the waves observed by Thome (1968), it would seem not beyond the realm of possibility that the earth's curvature can be an important contributing factor to F-region gravity wave ducting.

(6) "Potential" Model and the Hines Dispersion Relation.

In regions where the Brunt-Vaisala frequency is a slowly varying function of height, the "potential" model reduces to the Hine's theory for small horizontal phase velocity; (large λ_x^2). From equation (2) we see that if $\lambda_x^2 \gg |v|$, (2) reduces to

$$\frac{d^2\psi}{dz^2} + \left(\frac{\omega_b^2}{\omega^2} - 1\right) K_x^2 \psi = 0 \,. \tag{12}$$

Substitution of plane wave solution $e^{\pm iK_z z}$ in (12) will immediately yield the Hines dispersion relation.

Generally speaking, the "potential" is of the order of 25 $Km^{-2}sec^2$ which corresponds to a horizontal phase velocity of 200 m/sec. This means that for v_{phx} << 200 m/sec, the Hines model would be rather good in regions where ω_B is a slowly varying function of height.

To compare the various solutions we have shown a plot of the S_1 and S_2 modes together with the "free" gravity wave solution for horizontal phase velocities of 707.1 m/sec and 213.2 m/sec in Fig. 12. As can be seen, it is difficult to plot them all on the same scale. The "free" gravity wave corresponding to 213.2 m/sec is already approaching a simple sine wave. It is interesting to note that in the "potential" picture one sees immediately that all gravity waves with horizontal phase velocity higher than 310 m/sec must necessarily be F-region waves. They must all become evanescent below about 120 Km. The 707.1 m/sec gravity wave clearly illustrates this.

We have so far confined ourselves to investigating gravity waves with periods larger than 30 min. when the "potentials" do not vary significantly with frequency. For gravity waves with shorter periods than 30 min. the "potential" curves are drastically altered. Fig. 13 provides an example for a gravity wave with only a 7.5 min. period. The investigation of such types of waves is beyond the scope of the present report.

III. <u>On Non-Linear Effects of Gravity Waves</u>

There are many airglow oscillations showing a small period (8-14 minutes) oscillation superimposed over a long period oscillation of the order of hours. Most of the data available are given by the 5577 Å OI airglow emission and to a lesser extent the 6300 Å red line emission. Sample examples of such data will be given in the next section (Summary of preliminary work). Our aim here is merely

to state the principal objectives of our proposal. The airglow data may be summarized as follows.

(1) Most of the data are not tied to any known impulsive energy sources such as an explosion.

(2) Some of the data shows that the small period oscillation follows the long period oscillation in time sequence and is in fact slowly excited as the long period oscillation increases in intensity.

(3) There appears to be some variation in the period of the small period oscillation with the undulation of the long period oscillations.

There are theoretical analyses which may possibly be invoked to explain these results (although the original authors may not quite have had such a situation in mind when they proposed their theory), for instance the treatments of Row (1967) and Liu and Yeh (1971). Row's analysis deals with point impulsive sources. Liu and Yeh's treatment deals with a generalization of the point source to include spatial and temporal extensions. They were also able to show that at far distances ($\sim$ 1000 Km) away from the source we can obtain a short period Accoustic frequency oscillation superimposed over the long-period gravity-wave oscillation. While we have no doubt that their treatment is sound we do have some reservations over whether it can be used for explaining the observed results for the following reasons:

(1) These theoretical treatments do require some kind of impulsive energy sources.

(2) The results of Liu and Yeh (1971) show that at far distances from the source ($\sim$ 1000 Km) the short-period (accoustic frequency) wave arrives first followed by the long period gravity wave.

(3) In both these treatments an inviscid lossless atmosphere is assumed. If we were to use such a theory to explain the airglow oscillation we must hence assume that both the gravity wave and the accoustic frequency wave originate from far distances.

(4) The observed short period oscillation occurs only when the long period oscillation has a large amplitude. Liu and Yeh's (1971) treatment is essentially a linear treatment and does not allow a sharp cut-off of the short period oscillation at certain amplitudes in the long period oscillation.

We believe that the observed airglow oscillations just mentioned are typical examples of non-linear coupling between a gravity wave and the local atmosphere. We propose the following simple model which we intend to later develop into a full scale theory based on hydrodynamic equations.

Generally speaking, the hydrodynamic equations are coupled non-linear partial differential equations for dependent field variables such as pressure $p(\vec{x},t)$ density $\rho(\vec{x},t)$ and velocity $\vec{v}(\vec{x},t)$. From a mathematical point of view it is very difficult to handle non-linear partial differential equations and from a physical point of view it is also a lot easier to visualize a lump of fluid in oscillation rather than dealing with field variables. Our first step is therefore to go from a field picture into a particle (lumped mass) picture.

In general, if $f = f(z,t)$ is any field observable (f may be the pressure, density or temperature), $f(z(t,z_o),t)$ is the same observable of a particular element of fluid (a particle) identified by its initial position z_o at $t = 0$. Here $z = z(t,z_o)$ is of course the trajectory of the fluid element. It is possible to show that $\frac{\partial f\ (z,t)}{\partial t}$ or $\frac{\partial f\ (z,t)}{\partial z}$ may also go into their corresponding particle picture if we substitute $z = z(t,z_o)$ in $\frac{\partial f(z,t)}{\partial t}$ or $\frac{\partial f\ (z,t)}{\partial z}$. Furthermore, it makes no difference whether we substitute in $z(t,z_o)$ first and then differentiate or differentiate first and then substitute. In the former case $\frac{\partial f\ (z(t,z_o),t)}{\partial z}$ is just the change in f with respect to a change in the particle trajectory while keeping the time fixed. The same applies to higher order derivatives.

We shall for the present report neglect heat conduction and only use phenomenology for viscosity. We shall also assume that variations along the horizontal

directions may be neglected. It is then easy to show that the hydrodynamic derivative

$$\frac{Df}{Df}\bigg|_{z=z(t,z_o)} = \frac{\partial f}{\partial t} + z\,\frac{\partial f}{\partial z} \quad .$$

We will use the above procedure to convert Euler's field equation into a particle equation. As a first step, we will derive the equation for the natural oscillation of the atmosphere in an unperturbed hydrostatic background. The usual method used for such a derivation has always been within the framework of a particle picture in which the "particle" is an element of fluid contained in a weightless invisible plastic bag. In order to later incorporate the gravity wave field as a background in addition to the hydrostatic background and consider the coupling between the gravity wave and the Brunt oscillation, it is much more natural to derive the Brunt oscillation from a field picture.

The field equations can then be converted into "particle" equation which is just the equation for simple harmonic motion. We can then use very similar procedure to derive the Brunt oscillation with the gravity wave field as background. After conversion into a "particle" equation we would have a Hill's type of equation.

We will show that the time dependent parameter will have a fundamental frequency equal to the gravity wave frequency and that parametric excitation of the natural frequencies can occur. Since the velocity gradient term in the hydrodynamic equation is generally considered the most important non-linear term we will incorporate this into our original Euler's field equation. The term becomes a driving term for the "particle" equation (10) in the enclosed preprint.

Theory

We begin by considering the displacement of a fluid element from its equilibrium position z_o in a hydrostatic background atmosphere. We will neglect all variations along the horizontal plane. If we assume that the displaced fluid element is adiabatically compressed by the surrounding background atmosphere, we may write:

$$\rho(z) = \rho_o(z_o) + \frac{p_o(z) - p_o(z_o)}{c^2} = \rho_o(z_o) + \frac{\delta p}{c^2} \tag{1}$$

$$p(z) = p_o(z)$$

We may now substitute (1) into the Euler equation

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \tag{2}$$

when w is the vertical velocity field of the fluid.

If we expand $p_o(z)$ in (1) about z_o in Taylor Series, we obtain:

$$\delta p = p_o(z) - p_o(z_o) \sim \left.\frac{\partial p_o}{\partial z}\right|_{z_o} (z - z_o) = -g\rho_o(z - z_o)$$

In first order $(z - z_o)$, we may hence write,

$$\left[\rho_o(z_o) + \frac{\delta p}{c^2}\right]^{-1} \frac{\partial p}{\partial z} = \rho_o^{-1} \left[1 - \frac{\delta p}{\rho_o c^2}\right] \left[\left[\frac{\partial p_o}{\partial z}\right]_{z_o} + \left.\frac{\partial^2 p_o}{\partial z^2}\right|_{z_o} (z-z_o)\right]$$

$$\sim \frac{1}{\rho_o}\left.\frac{\partial p_o}{\partial z}\right|_{z_o} - \frac{\delta p}{\rho_o^2 c^2}\left.\frac{\partial p_o}{\partial z}\right|_{z_o} + \frac{1}{\rho_o}\left.\frac{\partial^2 p_o}{\partial z^2}\right|_{z_o} (z - z_o)$$

$$= \frac{1}{\rho_o}\left.\frac{\partial p_o}{\partial z}\right|_{z_o} - \frac{g^2}{c^2}(z - z_o) - \frac{g}{\rho_o}\left.\frac{\partial \rho_o}{\partial z}\right|_{z_o} (z-z_o)$$

$$= \frac{1}{\rho_o}\left.\frac{\partial p_o}{\partial z}\right|_{z_o} - \left[\frac{g}{\rho_o}\frac{\partial \rho_o}{\partial z} + \frac{g^2}{c^2}\right](z - z_o)$$

Hence, $$-\frac{1}{\rho}\frac{\partial p}{\partial z} - g = \left[\frac{g}{\rho_o}\frac{\partial \rho_o}{\partial z} + \frac{g^2}{c^2}\right](z - z_o)$$

We may write

$$\frac{Dw}{Dt} + \omega_B^2(z_o)(z-z_o) = 0 \tag{3}$$

where

$$\omega_b^2 = -\left[\frac{g}{\rho_o}\frac{\partial\rho_o}{\partial z} + \frac{g^2}{c^2}\right]$$

is the usual definition for the Brunt frequency.

We may now transform equation (3) into a "particle" equation by substituting the trajectory $z = z(t)$ into equation (3).

$$\frac{Dw(z(t),t)}{Dt} = \ddot{z}$$

The final "particle" equation becomes:

$$\frac{d^2}{dt^2}(z - z_o) + \omega_B^2(z_o)(z - z_o) = 0 \quad . \tag{4}$$

This is just the equation for the Brunt oscillation of a small fluid element about z_o. Normally equation (4) is derived with the assumption that the fluid is contained in a weightless, flaccid, thermal insulating bag. Such a derivation essentially assume a "particle" picture right from the beginning. We have formulated the problem and derived equation(4) from a field picture. This will allow us to generalize much more easily to the case when the background is no longer a simple hydrostatic atmosphere.

We shall now consider the case when the background consists of a linearized Hine's type of gravity wave propagating in an isothermal atmosphere. A fluid element in equilibrium with the background will no longer be stationary but will describe a simple harmonic trajectory given by:

$$z_o(t,z_o) = \frac{U_{z_o}}{\omega}\sin\omega t \tag{5}$$

where U_{z_o} is the amplitude of the gravity wave velocity field at z_o.

Once again we assume that if a fluid element is displaced from its equilibrium, it will be adiabatically compressed by the surrounding background which now consists of both the hydrostatic atmosphere and the gravity wave. Thus, we may write in place of equation (1)

$$\rho(z,t) = \rho_B(z_o(t)) + \frac{P_B(z) - P_B(z_o)}{c^2} = \rho_B + \frac{\delta P_B}{c^2}$$

(6)

$$p(z,t) = p_B(z_G(t))$$

where

$$\rho_B(z,t) = \rho_o(z) + \Delta\rho_G(z,t)$$

(7)

$$p_B(z,t) = p_o(z) + \Delta p_G(z,t)$$

and $\Delta\rho_G$ and Δp_G are the gravity wave density and pressure.

Since long period gravity waves generally have very long horizontal wave length, we shall again assume horizontal stratification and write the Euler equation as:

$$\frac{Dw}{Dt} = \frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad . \tag{8}$$

Substituting equation (6) into (7) we obtain:

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g = \frac{1}{\rho_B + \frac{\delta p_B}{c^2}} \quad \frac{\partial p_B}{\partial z} + g \tag{9}$$

$$= \left[\frac{1}{\rho_B} - \frac{1}{\rho_B}\ \frac{\delta p_B}{c^2}\ \frac{1}{(\rho_B + \frac{\delta P_B}{c^2})}\right] \frac{\partial p_B}{\partial z} + g.$$

Neglecting quadratic terms in δP_B, we obtain

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g \sim \left[\frac{1}{\rho_B}\ \frac{\partial p_B}{\partial z} + g\right] - \frac{1}{\rho_B^2}\ \frac{\delta P_B}{c^2}\ \frac{\partial P_B}{\partial z} \quad . \tag{10}$$

By making Taylor's series expansion about z_G, neglecting second order terms in $\Delta\rho_G$, ΔP_G, $(z - z_G)$ and using the following hydrostatic and gravity wave equations,

$$\frac{\partial P_o}{\partial z} = -g\rho_o$$

(11)

$$\frac{1}{\rho_o}\ \frac{\partial(\Delta p_G)}{\partial z} = -\frac{g\Delta\rho_G}{\rho_o} - \frac{\partial U}{\partial t}$$

we obtain

$$\frac{1}{\rho_B}\frac{\partial p_B}{\partial z} + g \sim -\frac{g}{\rho_B}\frac{\partial \rho_B}{\partial z}\bigg|_{z_G}(z - z_o) - \frac{\partial U}{\partial t}\bigg|_{z_G} - \frac{g^2}{c^2}(z - z_G) \quad . \tag{12}$$

Substituting equation (11) in equation (7) and transform the resulting field equations into a particle equation by letting $z = z(t)$, we obtain:

$$\ddot{z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + g = \ddot{z} - g\left[\frac{1}{\rho_B}\frac{\partial \rho_B}{\partial z}\bigg|_{z_G} + \frac{g}{c^2}\right](z - z_G) - \frac{\partial U}{\partial t}\bigg|_{z_G} = 0 \quad . \tag{13}$$

we may write (13) as:

$$\ddot{z} - \ddot{z}_G + \omega_H^2 (z - z_G) = 0 \tag{14}$$

where $\ddot{z}_G = \frac{\partial U}{\partial t}\bigg|_{z_G}$

$$\omega_H^2 = -g\left[\frac{1}{\rho_B}\frac{\partial \rho_B}{\partial z} + \frac{g}{c^2}\right] \sim \omega_B^2\{1 + K_z X_o \sin[\omega(t - t_o) - K_z(z_G - z_o)]\} \tag{15}$$

where $X_o = \frac{U_{z_o}}{\omega}$

ω_B = Brunt frequency at z_o .

Equation (14) is similar to equation(4) except z_G replaces z_o and ω_H replaces ω_B. Equation 914) is a Hills type of equation with ω the gravity wave frequency as the fundamental frequency for parametric excitation.

In deriving equation (14), we have assumed the Hines' linearized approximation,

$$\frac{\partial(\Delta p_G)}{\partial z} + g\Delta\rho_G = -\frac{\partial U_z}{\partial t}$$

where U_z is the Hines gravity wave velocity. According to Hines (1960), the most important non-linear effects arise from the velocity gradient terms $(\vec{U} \cdot \vec{\nabla})\vec{U}$ which is neglected in the above approximation. A more accurate equation would then be given by:

$$\frac{\partial(\Delta P_G)}{\partial z} + g\Delta\rho_G = -\frac{\partial U_z}{\partial t} - (\vec{U} \cdot \vec{\nabla})U_z = -\frac{DU_z}{Dt}$$

where ΔP_G and $\Delta\rho_G$ are no longer the Hines' solution but each must now contain some correction terms.

Equation (14) would now be replaced by:

$$\frac{d^2(z-z_G)}{dt^2} + \omega_H^2(z_G)(z-z_G) = -\frac{g}{c^2}(\dot{z}_G)^2 \quad . \tag{16}$$

The solutions to equation (16) and other discussions are given in the enclosed preprint (Tuan et al (1976)).

With the time dependent frequency ω_H given by equation (15), it can be shown (Hodges (1967)) that so long as $K_z X_o < 1$ instability will not occur and Richardson's number R_i will always be greater than $\frac{1}{2}$. Actually, we are in fact quite safe since Hines (1974) has shown that even the usual lower limit for stability given by $R_i \sim \frac{1}{4}$ is somewhat conservative.

As shown in Fig. 3 and 4 in the enclosed preprint (Tuan et al (1976)), "self-excitation" can occur even in the presence of considerable viscous damping. The energy for the excitation of the natural oscillations is actually fed in from the gravity waves.

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FIGURE LEGENDS

Fig. 1. Temperature profiles for the Summer Season with exospheric temperature ranging from $600^{\circ}K$ to $2200^{\circ}K$.

Fig. 2. "Potentials" for long period gravity wave and an exospheric temperature of $600^{\circ}K$. The dashed curve is for the Summer while the solid curve is for Winter.

Fig. 3. "Potentials" for long period gravity wave and an exospheric temperature of $1800^{\circ}K$. The dashed and solid curves are for the Summer and Winter Seasons respectively.

Fig. 4. "Potentials" for long period gravity wave and an exospheric temperature of $2200^{\circ}K$. The dashed and solid curves are for Summer and Winter Seasons respectively.

Fig. 5. "Potential" for a gravity wave with a 2-hour period and an exospheric temperature of $1500^{\circ}K$. The figure also shows the two resonance states corresponding to $\lambda^2 = 10.43\ sec^2/km^2$ (v_{phx} = 309.7 m/sec) and $\lambda^2 = 14.33\ sec^2/km^2$ (v_{phx} = 264.2 m/sec). These two states are the only guided or partially guided modes this "potential" will permit.

Fig. 6. Plot of phase shift as a function of the eigenvalue λ^2. The phase shift goes through a very sharp change of π ratians at $\lambda^2 = 10.43\ sec^2/km^2$ and $14.33\ sec^2/km^2$.

Fig. 7. Plot of the eigenfunction ψ_1 for the Lamb (S_1) mode.

Fig. 8. Plot of the eigenfunction ψ_2 for the S_2 mode.

Fig. 9a. Analytic temperature profile with a very sharp temperature drop at the base of the thermosphere.

Fig. 9b. The "potential" for the temperature profile of Fig. 9a showing a potential well with a minimum at about 140 Km.

Fig. 10. Gravity wave with a two-hour period and a horizontal phase velocity of 707 m/sec.

Fig. 11. F-region gravity wave being ducted by the earth's curvature. The strong forward tilt of the phase front is also shown in the figure.

Fig. 12. Comparison of different gravity wave states showing the S_1 and S_2 partially guided modes as well as the "free" gravity wave modes. (1) is the F-region mode with a horizontal phase velocity of 707.1 m/sec while (2) is a lower altitude "free" mode with a horizontal phase velocity of 213.2 m/sec.

Fig. 13. "Potential" for a gravity wave with only a 7.5 min. period.

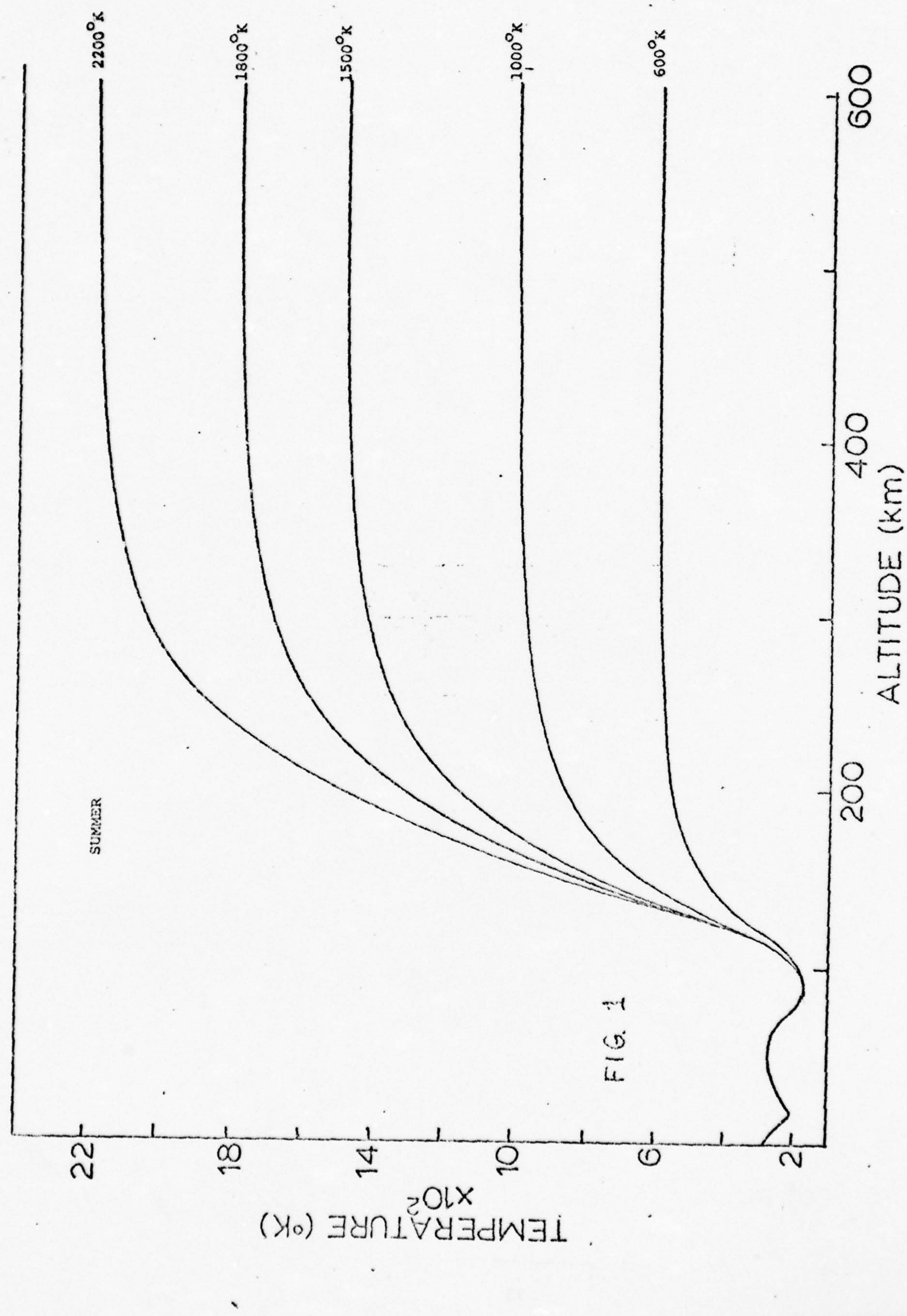
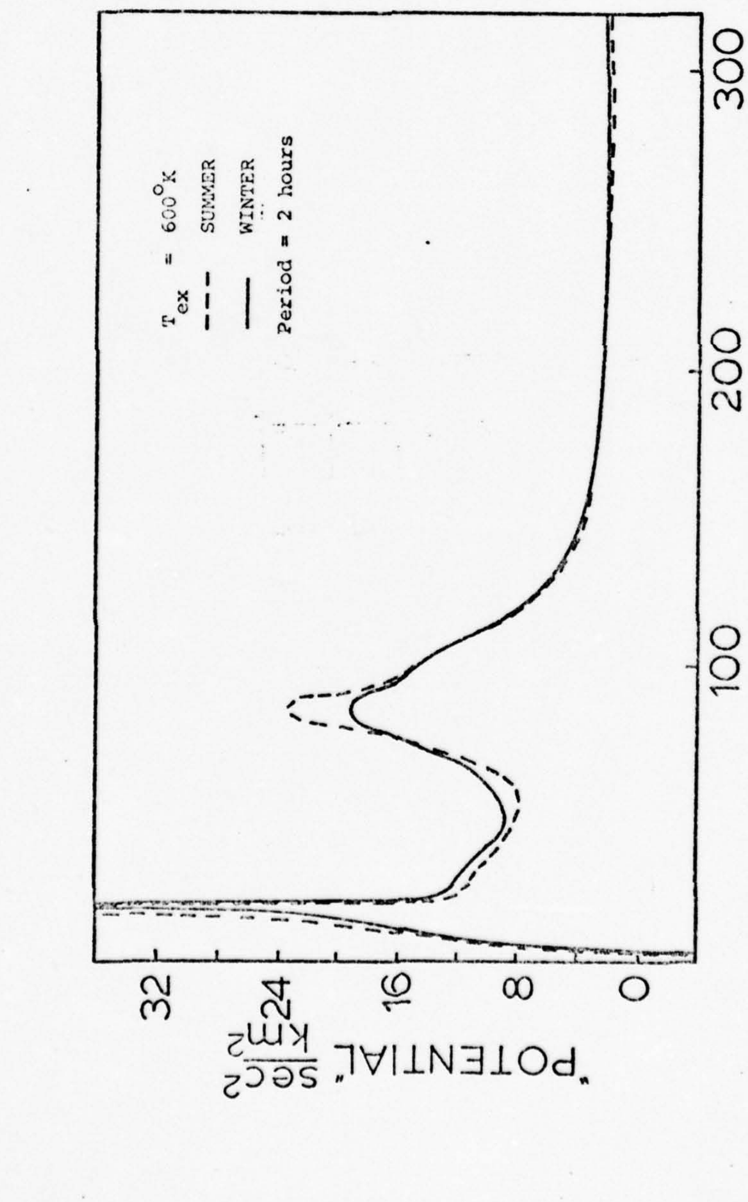

SUMMER
2200°K
1800°K
1500°K
1000°K
600°K
22
18
14
10
6
2
TEMPERATURE (°K) x10²
200
400
600
ALTITUDE (km)

FIG. 1



T_{ex} = 600°K
SUMMER
WINTER
Period = 2 hours
"POTENTIAL" $\frac{km^2}{sec^2}$
32
24
16
8
0
100
200
300
ALTITUDE (km)

FIG. 2

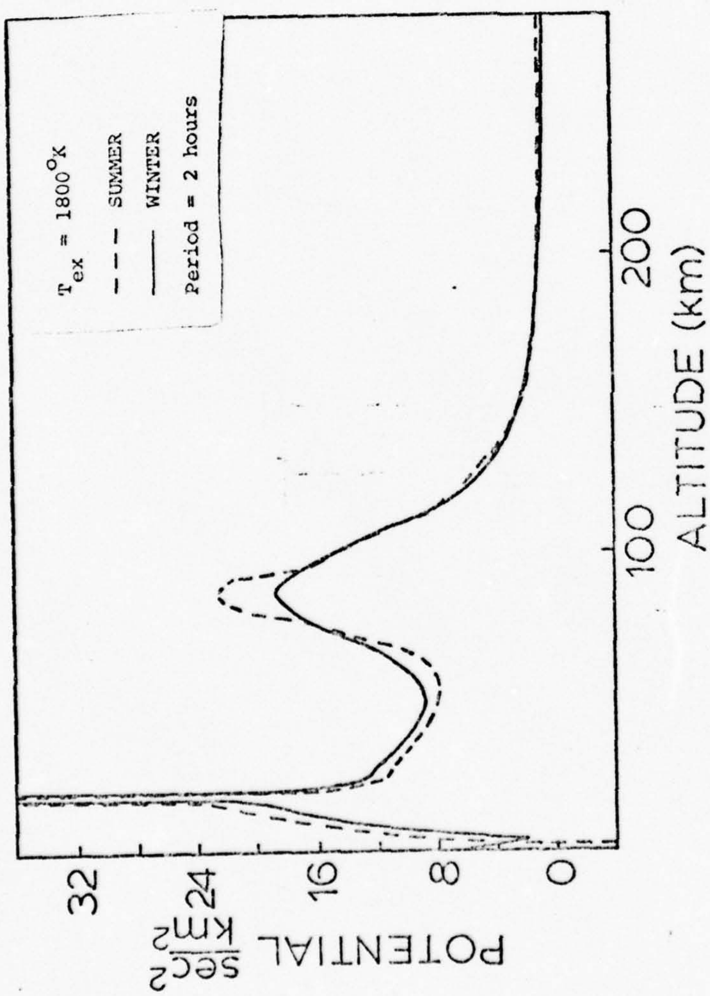

T_{ex} = 1800°K
SUMMER
WINTER
Period = 2 hours
POTENTIAL $\frac{km^2}{sec^2}$
32
24
16
8
0
100
200
ALTITUDE (km)

FIG. 3

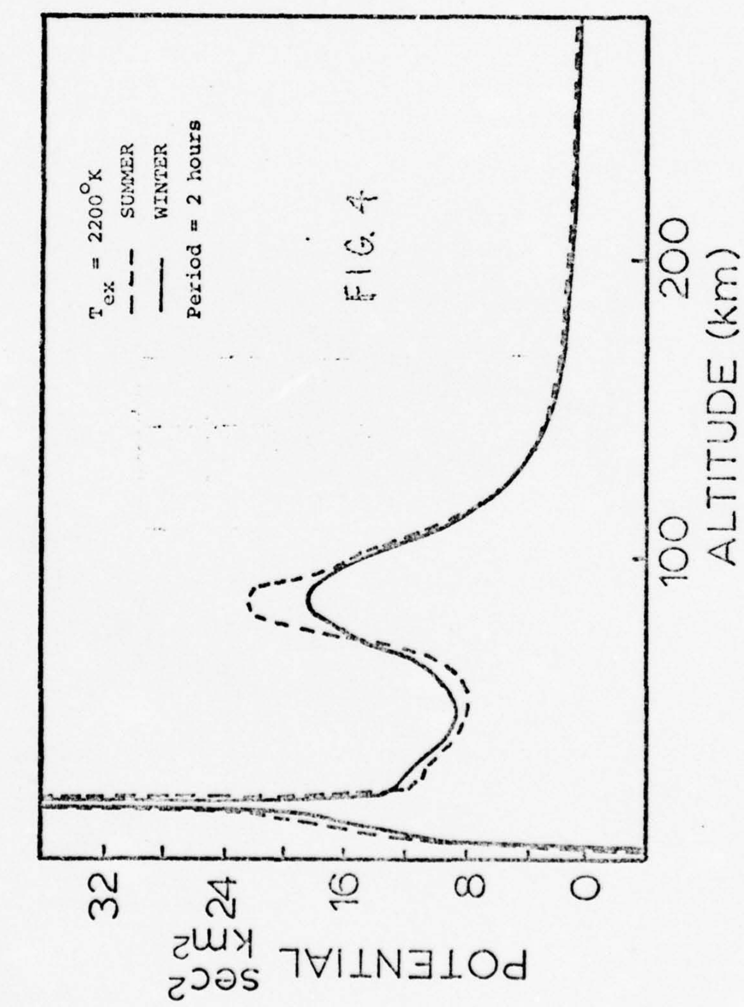

POTENTIAL $\frac{km^2}{sec^2}$
32
24
16
8
0
ALTITUDE (km)
100
200
T_{ex} = 2200°K
SUMMER
WINTER
Period = 2 hours

FIG. 4

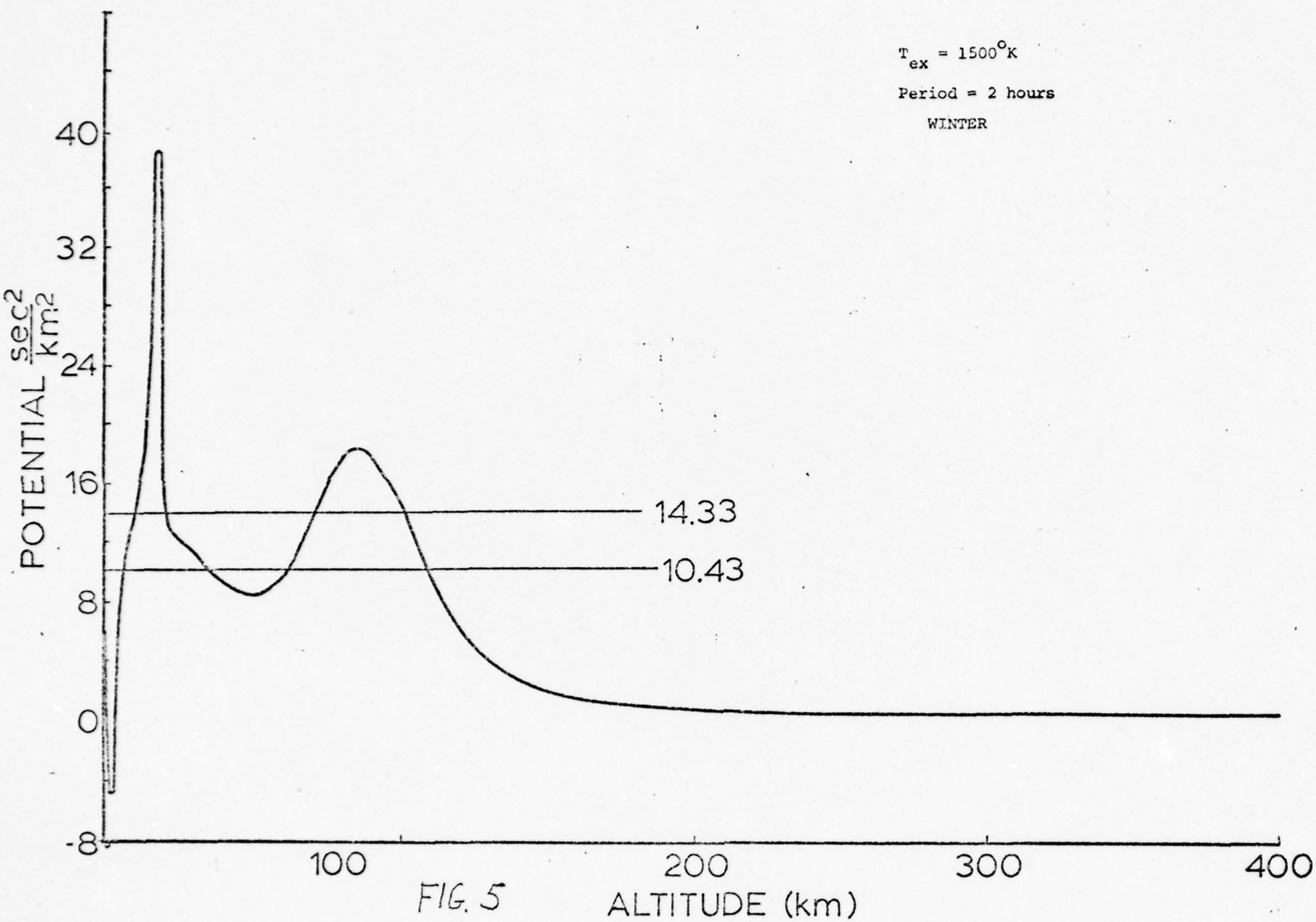

T_ex = 1500°K
Period = 2 hours
WINTER
POTENTIAL sec²/km²
40
32
24
16
8
0
-8
14.33
10.43
100
200
300
400
ALTITUDE (km)

FIG. 5

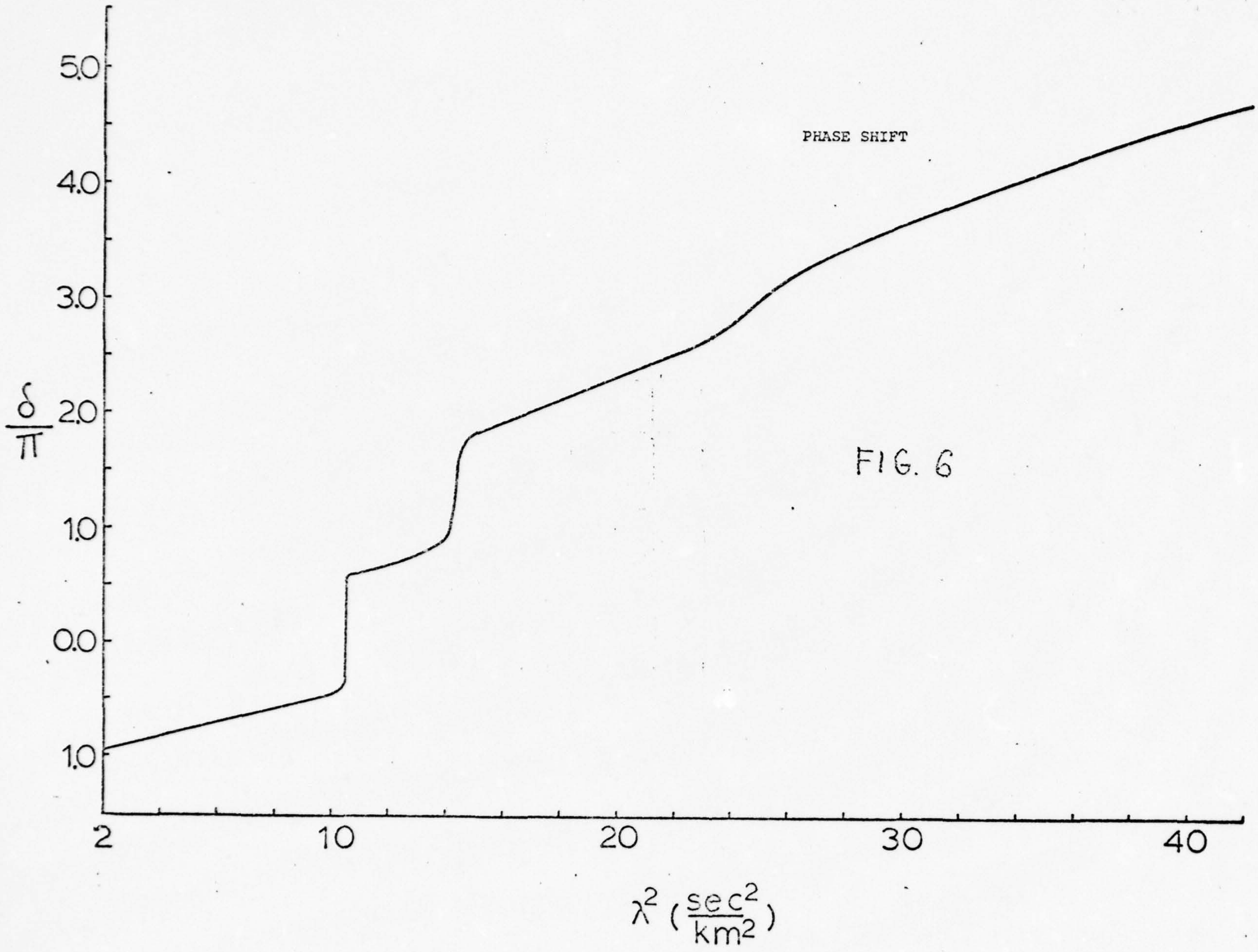

PHASE SHIFT
$\frac{\delta}{\pi}$
5.0
4.0
3.0
2.0
1.0
0.0
1.0
2
10
20
30
40
$\lambda^2 \left(\frac{sec^2}{km^2}\right)$

FIG. 6

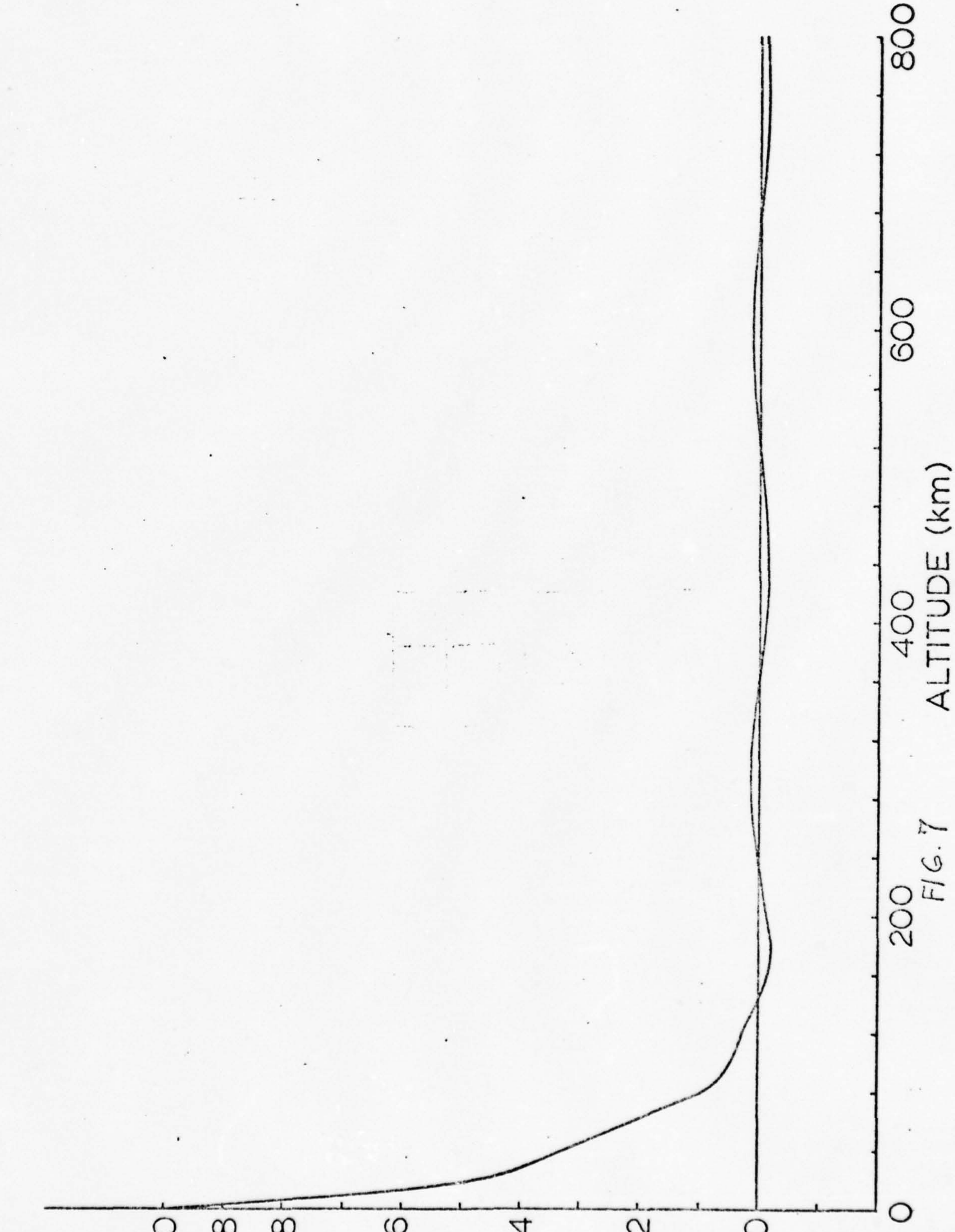


FIG. 7

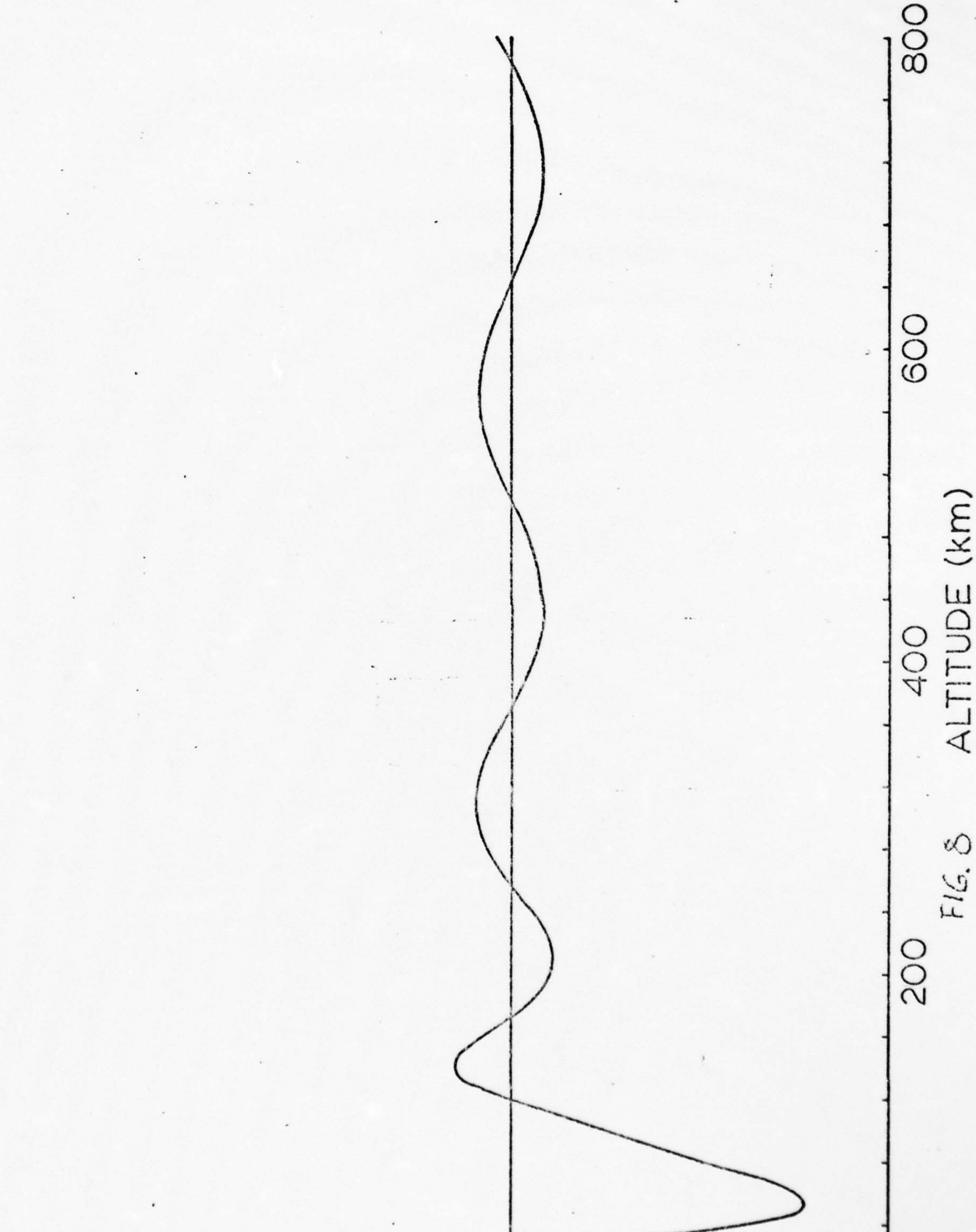

AMPLITUDE of S_2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
200
400
600
800
ALTITUDE (km)

FIG. 8

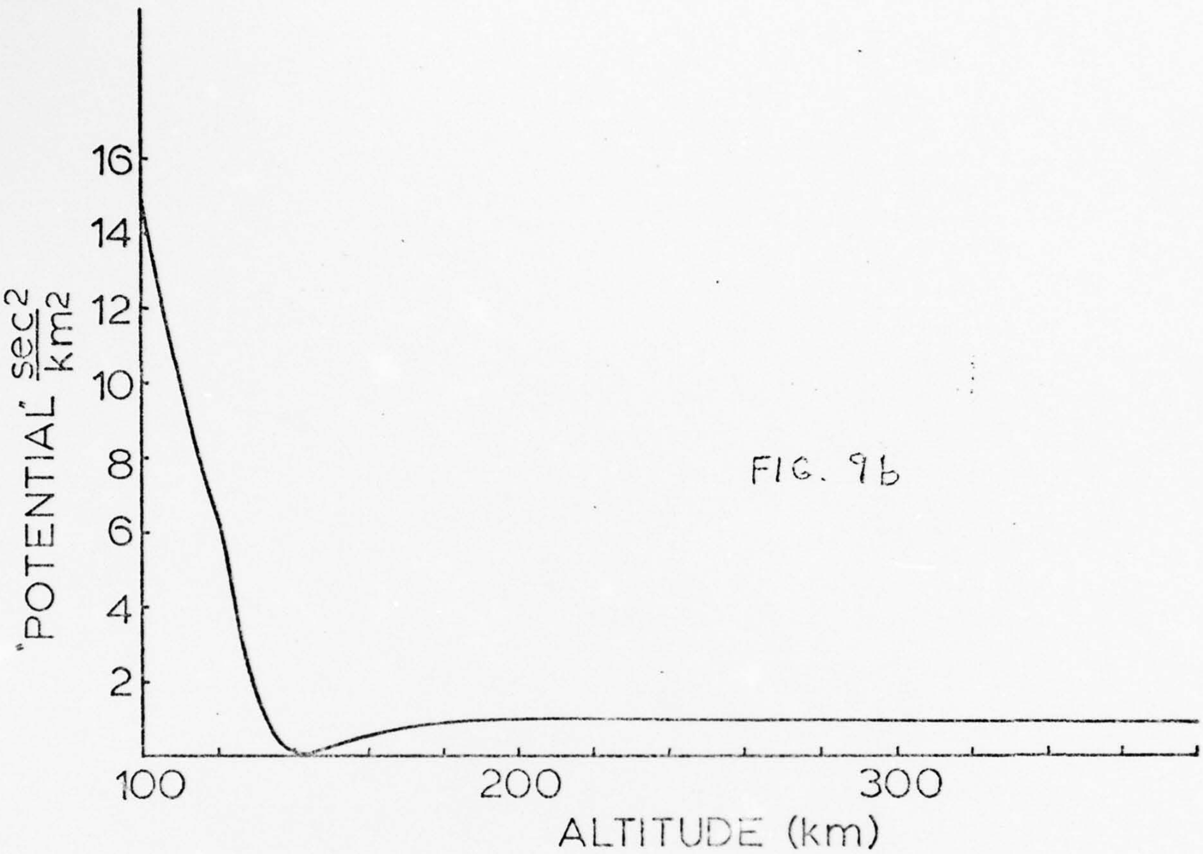

'POTENTIAL' $\frac{sec^2}{km^2}$
16
14
12
10
8
6
4
2
100
200
300
ALTITUDE (km)

FIG. 9b

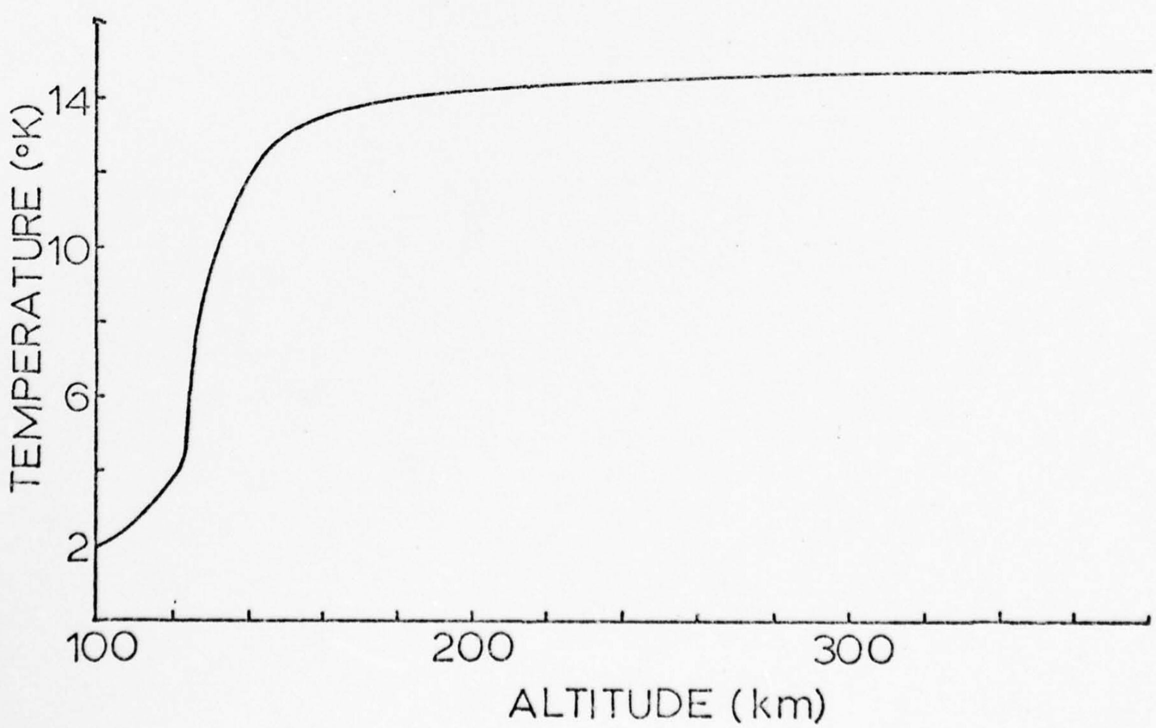

TEMPERATURE (°K)
14
10
6
2
100
200
300
ALTITUDE (km)

FIG. 9a

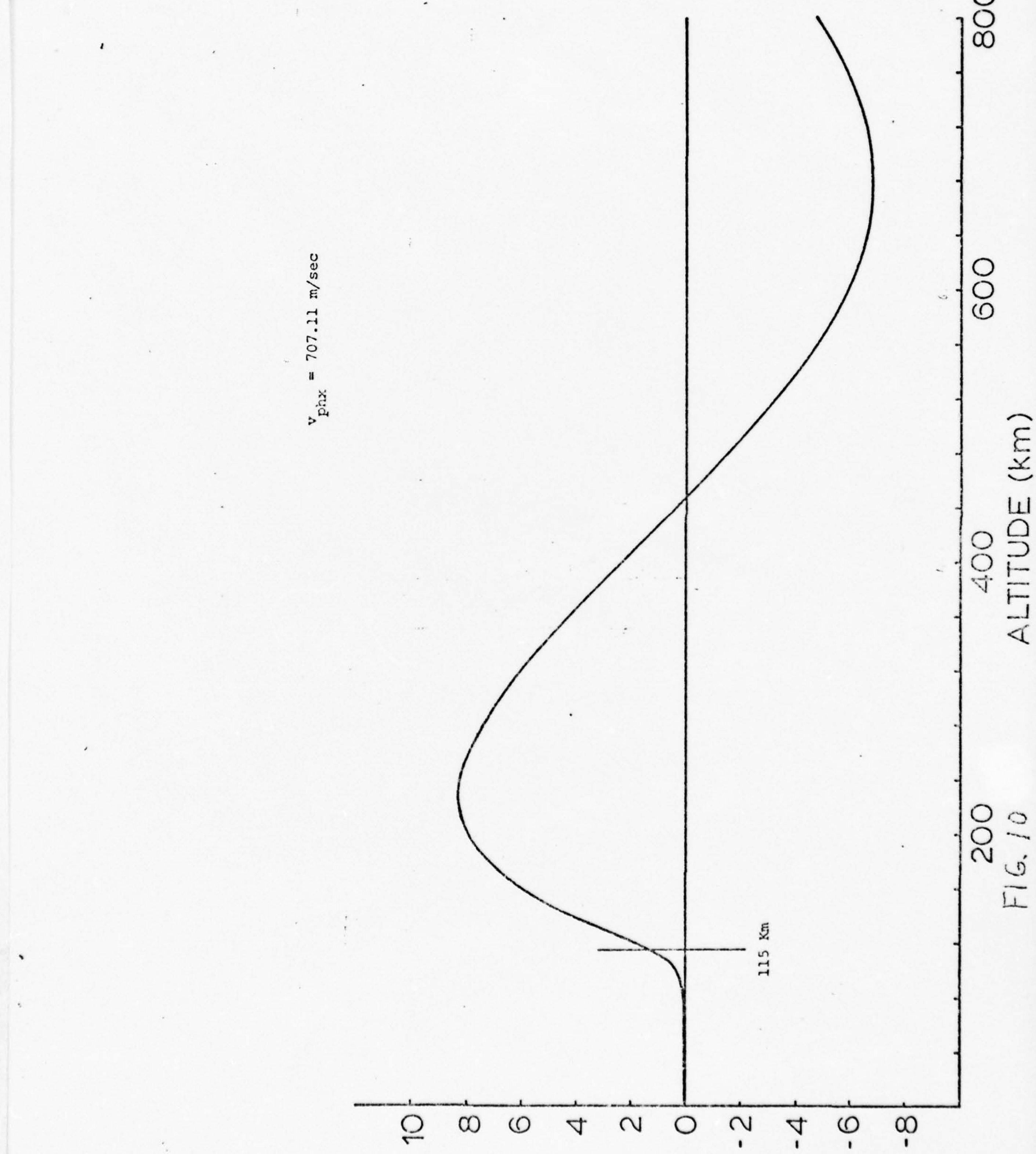

v_{phx} = 707.11 m/sec
115 Km
AMPLITUDE of Ψ
10
8
6
4
2
0
-2
-4
-6
-8
200
400
600
800
ALTITUDE (km)

FIG. 10

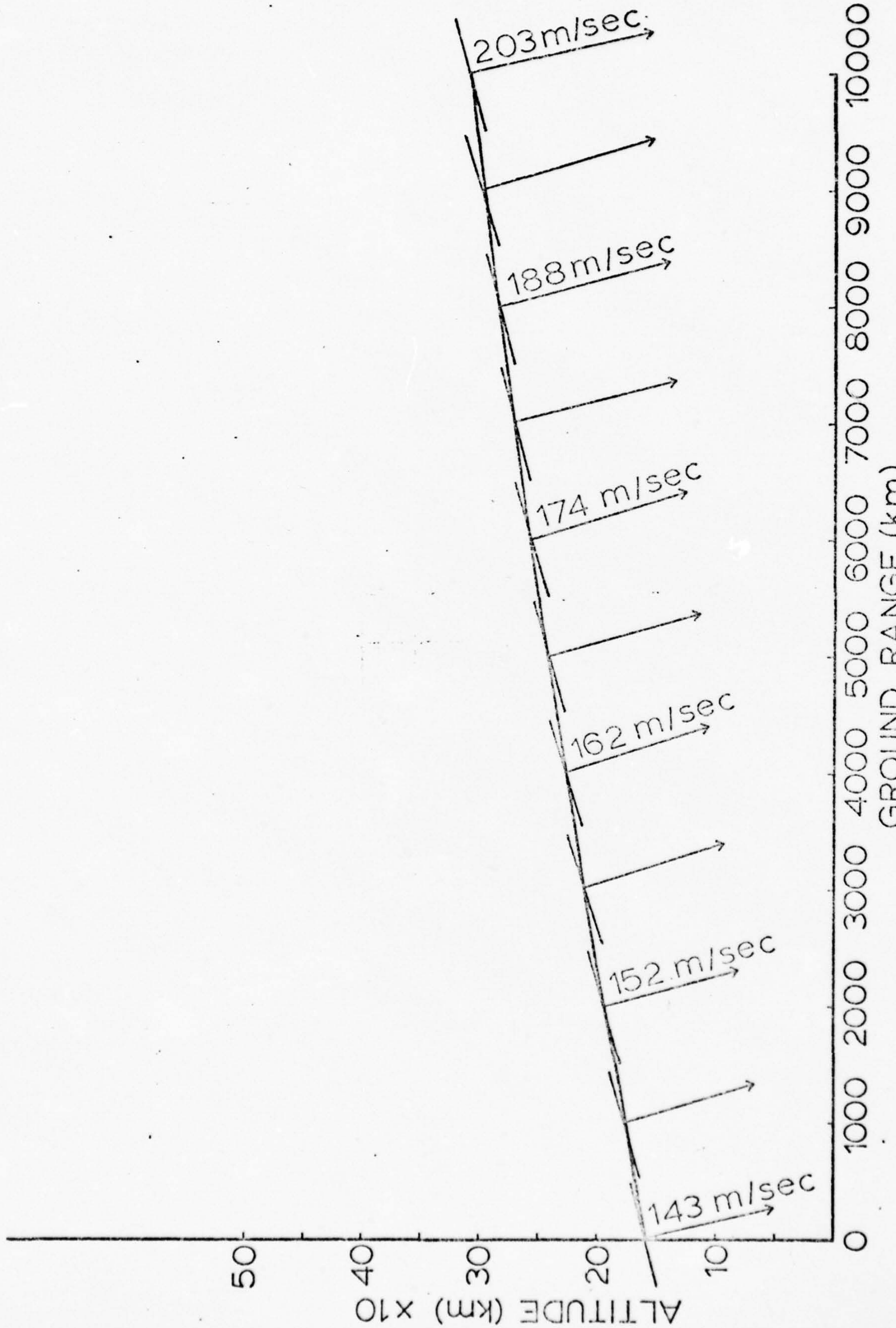
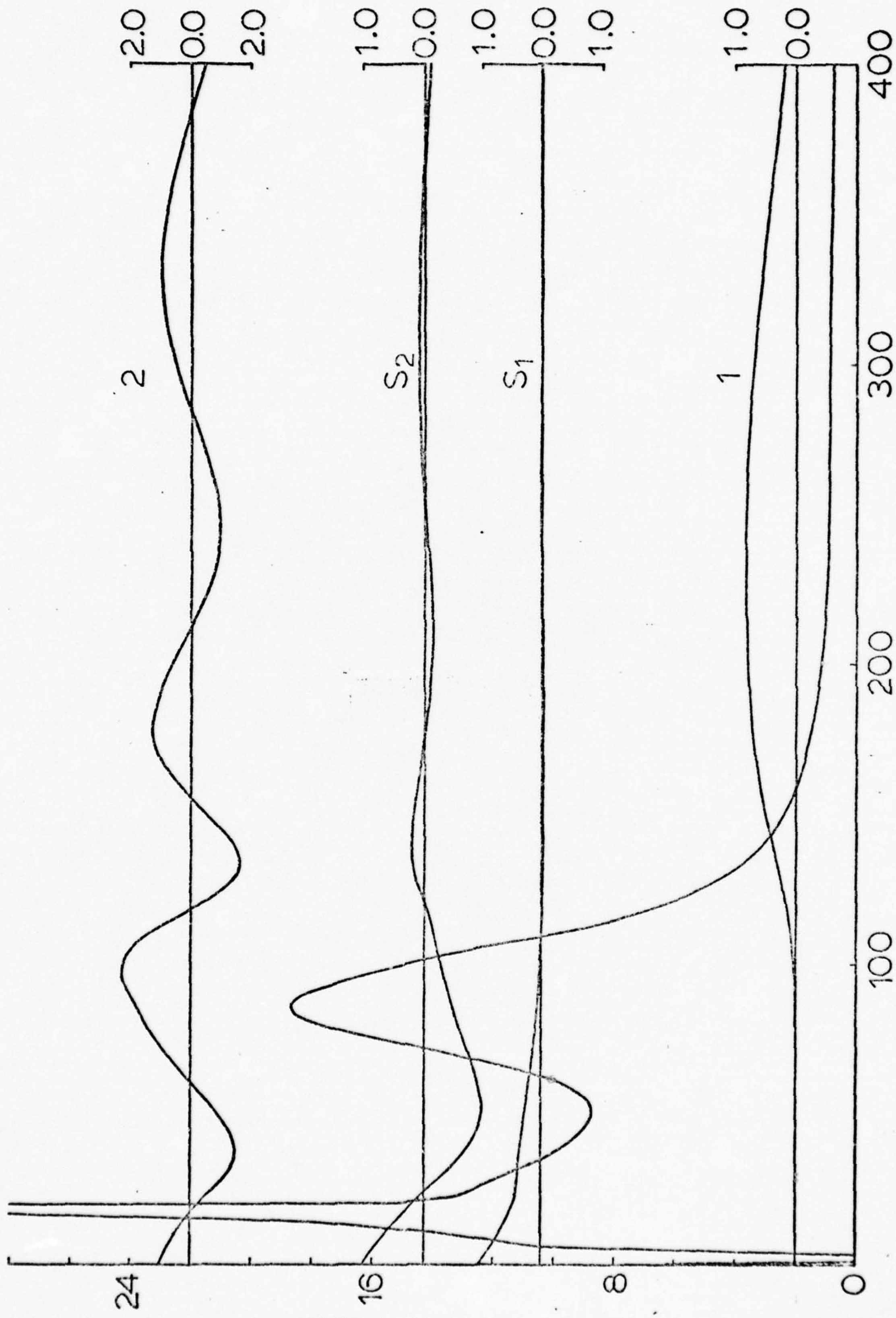

203 m/sec
188 m/sec
174 m/sec
162 m/sec
152 m/sec
143 m/sec
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
GROUND RANGE (km)
10
20
30
40
50
ALTITUDE (km) x10

FIG. 11



"POTENTIAL" $\frac{sec^2}{km^2}$
24
16
8
0
ALTITUDE (km)
100
200
300
400
2
S_2
S_1
1
2.0
0.0
2.0
1.0
0.0
1.0
0.0
1.0
1.0
0.0

FIG. 12

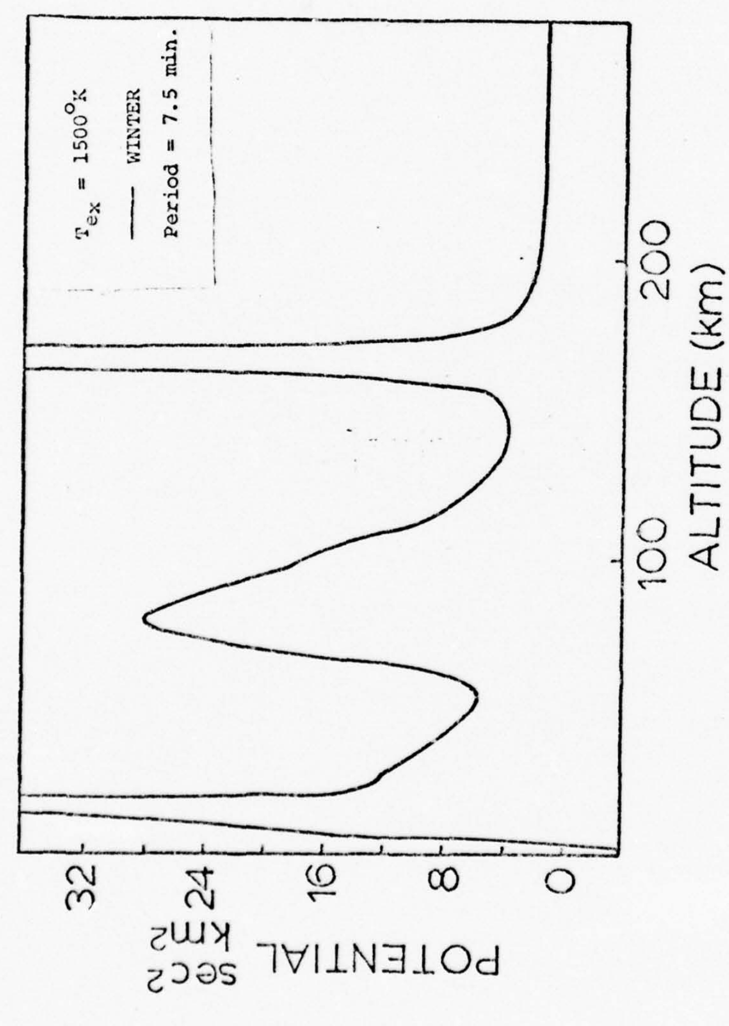

T_{ex} = 1500°K
WINTER
Period = 7.5 min.
POTENTIAL $\frac{km^2}{sec^2}$
32
24
16
8
0
100
200
ALTITUDE (km)

FIG. 13

"ON GRAVITY-WAVE INDUCED NATURAL OSCILLATIONS"

by

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"ON GRAVITY-WAVE INDUCED NATURAL OSCILLATIONS"

Abstract:

Airglow oscillations (mostly 5577 Å oxygen emission) which show a small period (8-14 min.) oscillation superimposed on a long period oscillation (several hours) have frequently been observed. Some data shows that the short period oscillation is "excited" by the long period oscillation as the latter increases in amplitude. The effects often occur in the absence of impulsive energy sources. We propose a model in which the short period oscillation is relatively localized and is associated with the natural (Brunt-Vaisala) oscillation of the atmosphere. This oscillation is triggered off by a long period gravity wave through non-linear coupling.

Introduction:

There exists a considerable body of airglow data showing a small (6-14 min.) period oscillation superimposed over a long period oscillation of the order of hours.

Based on assuming an inviscid, lossless and isothermal atmosphere, theoretical calculations have shown that an impulsive energy source such as, for instance, a bomb explosion can produce at far distances (say of the order of thousands of kilometers) a long period gravity wave together with a short-period acoustic frequency wave (Row, 1967, Liu and Yeh, 1971). However, the result requires the short period acoustic frequency wave to arrive first followed by the long period wave.

Much of the airglow data mentioned in the first paragraph are not related to any known impulsive energy sources. Furthermore, some of the data appears to show that the short-period oscillation actually follows rather than procedes the arrival of the long-period oscillation and is slowly excited as the long period oscillation increases in amplitude.

We propose to examine a model (Tuan et al., 1975) in which we consider the short-period oscillation to be local and to be associated with the natural Brunt-Vaisala oscillation of the atmosphere. The long period oscillation will be assumed to be a gravity wave which may be generated a long distance away. The non-linear part of the gravity wave serves as a forcing term which couples the B-V oscillation with the gravity wave. In the limit of small oscillation amplitudes, the gravity wave becomes linear and we have essentially damped simple harmonic motion. As the amplitude of the gravity wave increases, departure from linear behaviour occurs. If the amplitude of the gravity wave increases just slightly beyond a certain value, the non-linear term becomes strong enough to excite natural oscillation and we have a superposition of both the natural oscillation and the gravity wave oscillation.

Summary of Observed Results:

A typical superposition of a small period (∿ 8-10 min.) oscillation superimposed over a long period oscillation for the 5577 Å OI is shown in Fig. 1 (Okuda,

1975). The data is taken at Sendai in January 1966.

Fig. 2(a) (Okuda, (1962), Silverman (1962)) shows the interesting case where the long period oscillation increases in intensity producing the "self excitation" in the short period oscillation. To a much lesser extent, the Oct. 28-29 (1961) result of Fig. 2(b) also shows this effect. By contrast, on a relatively calm day (Oct. 21-22) there is relatively much less oscillation. The very gradual nature of the introduction of the short period oscillation which essentially follows rather than precedes the long period one seems to suggest that the excitation is local and most certainly not impulsive.

Theory:

In the absence of gravity waves, a vertical displacement of a fluid element will immediately bring about a restoring buoyancy force. For small displacements, neglecting viscosity, simple harmonic motion is produced. It is given by:

$$\frac{d^2(z - z_o)}{dt^2} + \omega_B^2(z_o)(z - z_o) = 0 \tag{1}$$

where $\omega_B^2 = -g\left[\frac{1}{\rho_o}\frac{\partial \rho_o}{\partial z} + \frac{g}{c^2}\right] = \frac{(\gamma - 1)g^2}{c^2} + g\frac{c^{2'}}{c^2}$

$\rho_o = \rho_o(z)$ is the background density

$z_o =$ the position at which the fluid element is in equilibrium with the background.

Equation (1) may be obtained directly from the Euler equation if we assume

$$\rho(z) = \rho_o(z_o) + \frac{P_o(z) - P_o(z_o)}{c^2} \tag{2}$$

$$P(z) = P_o(z) = \text{background pressure} \tag{3}$$

Equation (2) gives the density of the fluid element when it is displaced slightly from z_o to z and adiabatically compressed by the background pressure change.

In the presence of a gravity wave, we may assume that the background now consists of both an isothermal hydrostatic atmosphere and the gravity wave. Thus,

we may write

$$P_B(z,t) = P_o(z) + \Delta P_G(z,t) \tag{4}$$

$$\rho_B(z,t) = \rho_o(z) + \Delta\rho_G(z,t) \tag{5}$$

where ΔP_G and $\Delta\rho_G$ are the gravity wave pressure and density. If a fluid element is displaced from $z_G(t)$ to z, its density becomes:

$$\rho(z,t) = \rho_B(z_G(t)) + \frac{P_B(z) - P_B(z_G)}{c^2} \tag{6}$$

$$P(z,t) = P_B(z_G(t))$$

where $z_G = z_G(t,z_o)$ is the trajectory of the fluid element if it remains in equilibrium with the background. Substituting equation (4),(5),(6) and (7) into the Euler equation, we obtain, after suitable expansion, the following:

$$\frac{d^2(z - z_G)}{dt^2} + \omega_H^{\;2}(z_G)(z - z_G) = 0 \tag{8}$$

where

$$\omega_H^{\;2}(z_G) = -g\left[\frac{1}{\rho_B}\;\frac{\partial\rho_B}{\partial z} + \frac{g}{c^2}\right]$$

$$\sim \omega_B^{\;2}\{1 + k_z X_o \sin[\omega(t - t_o) - k_z(z_G - z_o)]\} \tag{9}$$

In equation (9), k_z is the vertical wave number, $z_G = X_o \sin \omega t$ and X_o is the amplitude of the gravity wave oscillation. It is easy to show that $\omega_H^{\;2}$ given by equation (9) is identical to the expression given by Hodges (1967) for $\frac{2k_z\mu T_o}{g} >> 1$. We will consider for the rest of this paper the special case where $k_z X_o < 1$. This means that the Richardson's number will always be greater than 1/2 so that instability will not occur, (Hodges, 1967).

In deriving equation (8) we have assumed the Hines' linearized approximation,

$$\frac{\partial(\Delta P_G)}{\partial z} + g\Delta\rho_G = -\frac{\partial U_z}{\partial t}$$

where U_z is the Hines gravity wave velocity.

According to Hines (1960), the most important non-linear effects arise from

the velocity gradient term $(\vec{U} \cdot \vec{\nabla})\vec{U}$ which is neglected in the above approximation. A more accurate equation would thus be given by:

$$\frac{\partial(\Delta P_G)}{\partial z} + g\,\Delta\rho_G = -\frac{\partial U_z}{\partial t} - (\vec{U} \cdot \vec{\nabla})U_z = -\frac{DU_z}{Dt}$$

where ΔP_G and $\Delta\rho_G$ are no longer the Hines' solution but each must now contain some correction terms.

Equation (8) would now be replaced by:

$$\frac{d^2(z - z_G)}{dt^2} + \omega_H^{\ 2}(z_G)(z - z_G) = (\vec{U} \cdot \vec{\nabla})U_z = -\frac{g}{c^2}(\dot{z}_G)^2 \qquad (10)$$

Equation (10) reduces to equation (8) for small gravity wave amplitudes. We would like to emphasize that while we have not specifically included viscosity effects in our discussion, we did include them phenomenologically in our calculations. They are not important and serve merely to damp out natural oscillations arising from transients. We shall leave them together with a more complete derivation of equation (10) for later.

Results:

Figures (3) and (4) are the theoretical results. We have assumed a Brunt Period of 4.9 minutes and an isothermal background atmosphere. Figure (3) shows what happens after the transients have been damped out by viscosity. The amplitude X_o of the gravity wave is 1.58 Km. Even with this much amplitude, the Richardson's number is still greater than 1/2 so instability will not occur. However, some significant "self-excitation" of the natural oscillation due to the non-linear forcing term occur. The period of such oscillations is of the order of 5 minutes close enough to the B-V oscillation.

Figure (4) shows what happens when we use heavy viscous damping. In general viscous damping depends on the size of the displaced fluid element. The smaller the fluid element, the greater the damping effect becomes. In Fig. (4) we see that

the transient effects are damped out in about one cycle and the "self excitation" begins after about 3 cycles of the gravity wave.

Discussion

While we do not claim that the superposition of a small period oscillation of the order of B-V frequency and a longer period gravity wave oscillation is always caused by non-linear coupling, the general shape of the oscillation curves and the manner in which the natural oscillation is excited do suggest that this may well be a plausible mechanism. We wish to emphasize that it is not surprising that "self excitation" can occur since equation (8) which is rigorously derived is essentially a Hills type of equation whose solution can be unstable. In fact, any disturbance introduced into equation (8) would produce natural oscillations which amplify with time and is only prevented from rapid increase by viscous damping. The energy is fed into the natural oscillation from the gravity wave. Unlike equation (8), the derivation of equation (10) involves a number of assumptions and must hence be considered as semi-phenomenological. All we wish to show here is that a $(\vec{U} \cdot \vec{\nabla})\vec{U}$ type of non-linearity is sufficient to generate natural oscillations even when Richardson's number is still relatively large.

ACKNOWLEDGEMENT

We much appreciate helpful discussions with Professor Y.G. Tsuei and some helpful computer advice from Professor R.H. Raible. A few important suggestions from Professor C.O. Hines over the telephone is gratefully acknowledged.

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Figure Captions

Figure 1: The 5577 Å OI emission is plotted against local time. The data is taken at Sendai by Okuda.

Figure 2: (a) This plot of 5577 Å OI emission provides a good example showing how a small period oscillation may be slowly excited by the long period gravity wave. (b) This shows a similar but less obvious self excitation at Sacremento Peak. A comparison is also made with a quiet day.

Figure 3: A small amplitude gravity wave with a one hour period is shown oscillating about 95 km. The wave approaches a sinusoidal shape for small enough amplitudes. The natural oscillation is damped out since for small amplitudes equation (10) reduces to equation (8).

Figure 4: Gravity wave with a period of one hour but with 10 times the damping (a much smaller fluid element) is shown in Fig. 4. The transient natural oscillation is damped out after one hour. Self excitation of natural oscillation began after 3 hours with much less oscillation amplitude than shown in Fig. 3. The present example is an extreme case.

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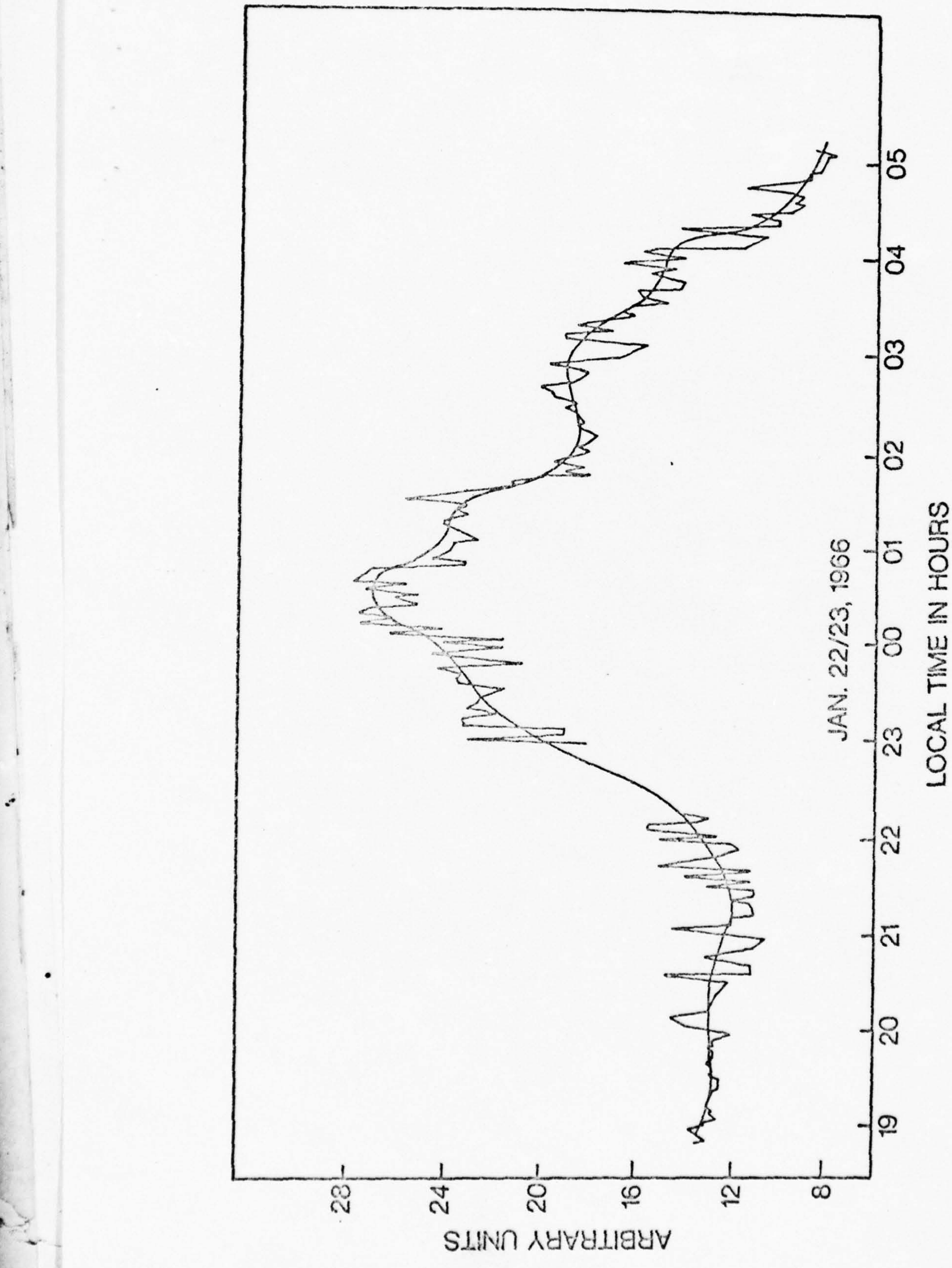
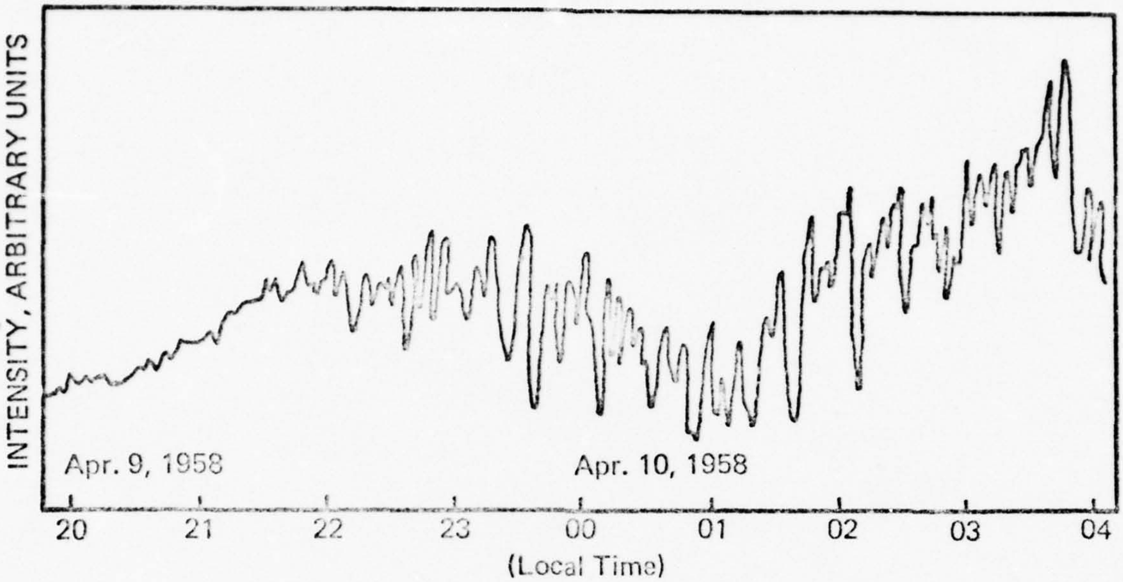
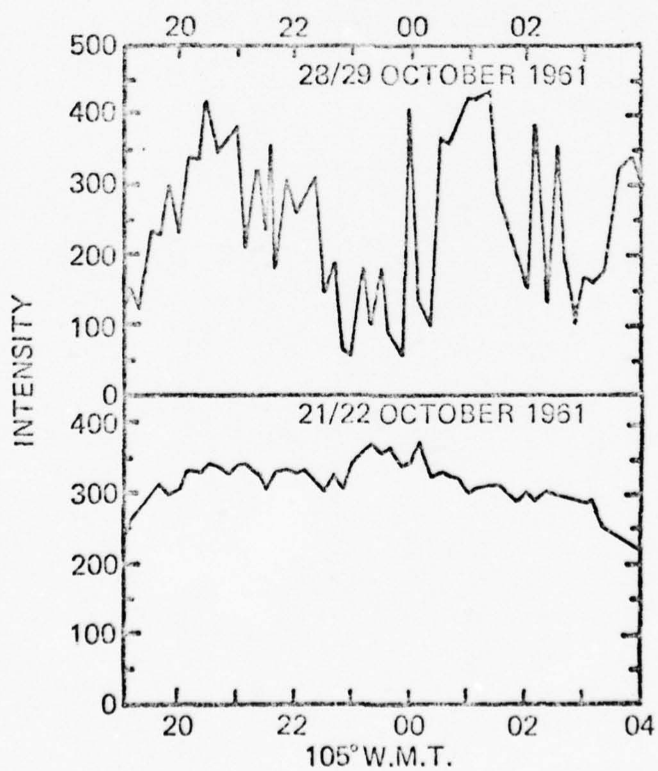

JAN. 22/23, 1966
ARBITRARY UNITS
28
24
20
16
12
8
LOCAL TIME IN HOURS
19
20
21
22
23
00
01
02
03
04
05

Fig. 1

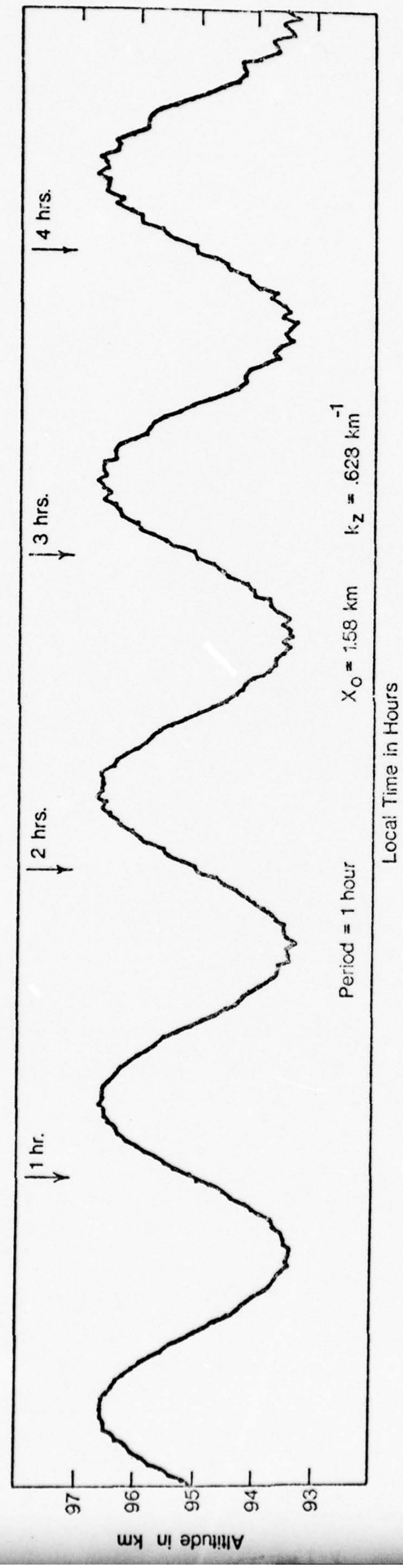


INTENSITY, ARBITRARY UNITS
Apr. 9, 1958
Apr. 10, 1958
20
21
22
23
00
01
02
03
04
(Local Time)



20
22
00
02
500
400
300
200
100
0
28/29 OCTOBER 1961
INTENSITY
21/22 OCTOBER 1961
400
300
200
100
0
20
22
00
02
04
105° W.M.T.

Fig. 2



Altitude in km
97
96
95
94
93
1 hr.
2 hrs.
3 hrs.
4 hrs.
Period = 1 hour
X_o = 1.58 km
k_z = .623 km^{-1}
Local Time in Hours

Figure 3

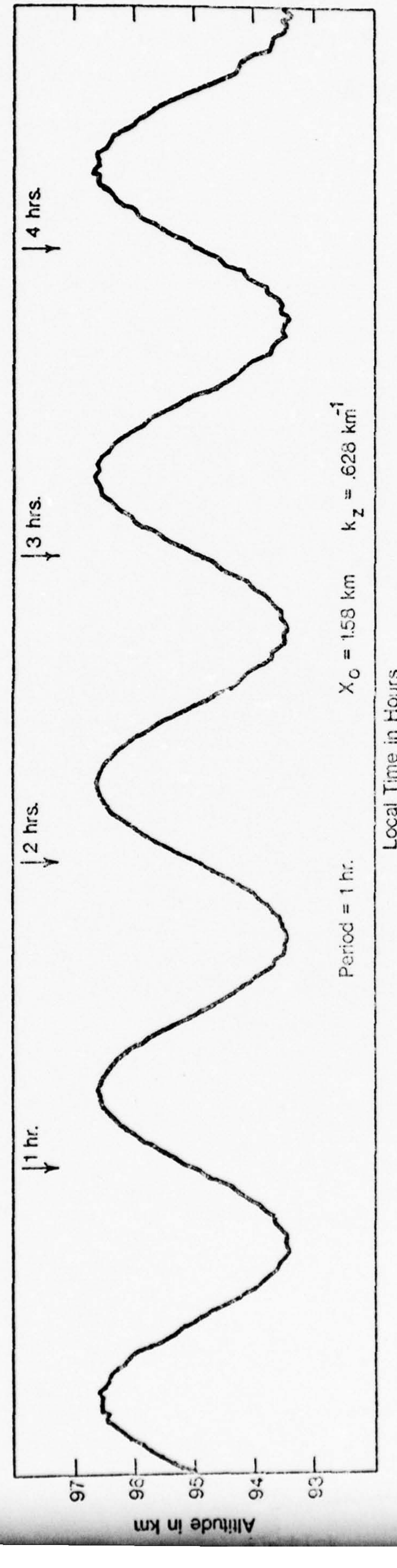

Altitude in km
97
96
95
94
93
1 hr.
2 hrs.
3 hrs.
4 hrs.
Period = 1 hr.
X_o = 1.58 km
k_z = .628 km^{-1}
Local Time in Hours

Figure 4

On The Behavior Of Airglow Under The Influence Of Gravity Waves

by

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Abstract

While the experimental observations of airglow intensity fluctuations and its possible connections with TID have been made by many investigators, very little attention has been given to a theoretical analysis of such ionospheric disturbances on airglow. In this paper we specifically examine the effect of an atmospheric gravity wave on the 6300 Å OI and the 5200 Å NI emission in mid-latitudes. We will also show that for the 6300 Å OI, if we assume reasonable values for the ionospheric parameters as well as a gravity wave with a period of approximately 3 hours, the resultant fluctuations in airglow is consistent with Dach's observed data as well as with the simultaneously measured variations in the ionospheric parameters.

Introduction

An association of airglow intensity variations with traveling ionospheric disturbances has been established by Barbier (1964, 1965) for the 6300 Å OI emission, and by Weill and Christophe-Glaume (1967) for both the 6300 Å OI and 5200 Å NI emissions by a direct comparison of the two types of measurements. In addition, some published data (Silverman, 1962; Okuda, 1962; Dachs, 1968) can, in retrospect, be seen as having been probably due to similar causes. The data of Dachs include observations on both 5577 Å OI and 6300 Å OI as well as of the ionospheric parameters foF2 and h'F2, and are thus particularly valuable for analysis from the point of view of a possible wave origin of the variations. Disturbances in airglow can, of course, arise from a variety of causes, and simultaneous measurements of other parameters are needed to establish the cause of a disturbance.

Over the past few years considerable theoretical and experimental work has been carried out on the effects of atmospheric gravity waves on ionospheric parameters. The effects of these waves on optical emissions, however, has received no attention from a theoretical viewpoint. A suitable analytic model for such a treatment has been presented recently by Porter and Tuan (1973). This model for the response of the F-layer to gravity waves differs fundamentally from the usual theoretical treatment in several respects. Of these the most pertinent for airglow applications are: (1) The analytic model is fully time-dependent. One result of such a model is that given a sinusoidal neutral gravity wave, the response of the electron density oscillation is no longer in general sinusoidal and the time behavior varies with height. This makes it more suitable

for studying both the general decay of the F-layer and airglow at night as well as such detailed behavior as differences in phase between the oscillations of electron density and the airglow. (2) The analytic model remains valid even if the percentage oscillations in the electron density or airglow is large compared with ambient values. This is especially important for the 6300 Å OI emission whose luminosity profile peaks at a position (about one scale height below F2 peak) where the electron density and airglow oscillations are rather large.

In this paper we first outline the computation for the 6300 Å OI emission intensity during the passage of a gravity wave. For long period gravity waves the continuity equation from which the emission intensity of the 6300 Å OI can be calculated reduces to the quasi equilibrium theories of Peterson, Van Zandt and Norton (1966), in which the production of the excited $O(^1D)$ states is assumed to equal the loss through optical emission and quenching. Comparison of the theoretically calculated changes in both electron density and luminosity profiles will serve to bring out the essential features of the theory.

We shall next consider the time behavior of the total electron content as compared with the airglow oscillations. It will be shown that the total electron content shows less oscillatory behavior, and that only by plotting the time derivative of the total electron content can the oscillations be more significant. However, such fluctuations are detectable (Davis and daRosa, 1969). The reduced oscillation in the total electron content, coupled with the large otical emission oscillations, provides us with a possible criteria for distinguishing between long period gravity wave effects and those produced by such other sources such as particle participation.

We will then consider the effect of a gravity wave on the 5200 Å NI emission intensity. Here we can no longer use the equilibrium theory that we have been using for the 6300 Å line, since, at least at higher altitudes, the production of $N(^2D)$ states can no longer be balanced by either the radiative loss or the quenching. We will also consider the importance of electron quenching, which is not significant for the 6300 Å OI line, but, as we shall see, is much more important for the nitrogen line.

The theory derived here will then be used for the interpretation of the experimental results of Dachs (1968) at Tsumeb, South West Africa. These include simultaneous measurements of foF2, h'F2, and the airglow intensity variations with time, thus allowing for a test of both aspects of the theory.

Finally, we shall also study the effect of varying the quenching rates on the phase of the oscillations. We will consider this phase difference in relation to the effect of other mechanism which can also cause a phase difference.

Time behavior of electron density and 6300 Å OI luminosity profiles

Theoretical treatments of the 6300 Å OI emission taking account of different factors and calculating various parameters are available from a number of authors (see, for example, Chamberlain, 1958; Lagos, Bellew and Silverman, 1963: Lagos, 1964; Peterson, Van Zandt and Norton, 1966; and Tuan, 1969). The following treatment leans most heavily on the work of Peterson, Van Zandt and Norton.

We make the following assumptions: (for their justification see Peterson, Van Zandt and Norton 1966); (1) the time derivative term in the continuity equation for the density of oxygen atoms in their $O(^1D)$ states may be neglected; (2) the diffusion term is neglected; and (3) we assume chemical equilibrium for the production and loss of O_2^+ molecules as well as neglecting the O_2^+ and NO^+ molecules in the equation for charge neutrality. We then obtain the following well known expression for the volume emission rate ε_{6300} for the 6300 Å line,

$$\varepsilon_{6300}\,(z,t) = \eta\left(\frac{A_{6300}}{A_{^1D}}\right)\frac{\gamma\left[O_2\right]N_e}{1+\frac{1}{A_{^1D}}\left(S_1\,[N_2]+S_2N_e\right)} \qquad (1)$$

where η is the efficiency for recombination, γ is the rate coefficient for charge transfer between O and O_2^+, S_1 and S_2 are the nitrogen and electron quenching coefficients respectively. O_2^+ profiles obtained by Schunk and Walker (preprint) could later be used in equation (1). However, since the same chemical equilibrium and charge neutrality approximations have been used in the expression for N_e, (Porter and Tuan, 1974), we have decided to maintain consistency and use equation (1) for the emission rate. Since our primary interest is in the basic

dynamic response of the airglow to gravity waves rather than any detailed behavior, we have also neglected the contribution from dissociative recombination of NO^+ (Silverman, 1970) as well as cascading from the ^1S states in the production of ^1D states (Rishbeth and Garriott, 1969). We will also neglect the time variation in O_2 and N_2 caused by the gravity waves. In reality the equations for such neutral atmospheric constituents are really coupled to the equation for the ion-electron plasma. To simplify all these processes we have used stationary models for O_2 and N_2. In practice, the approximations should be good since, as already mentioned, the percentage variation in the neutral gas caused by the gravity wave is usually very much less than the percentage variation for the ion-electron plasma, especially in important regions such as the peak of the luminosity profile. By far the most important contribution to the oscillation in the airglow arises from changes in the electron density N_e in equation (1).

Most recent theoretical treatments for the time-dependent behavior of F-layer electron density (e.g. Testud and Francois, 1971; Klostermeyer, 1972 a,b) under the influence of gravity waves are essentially based on perturbation theory in which the unperturbed ionosphere is assumed to be a stationary Chapman function independent of time. Also, in perturbation treatments the time dependence for the plasma is assumed to be sinusoidal if the gravity wave is sinusoidal, and a simple product form for the time and spatial dependence is usually assumed.

The luminosity peak for the night-time 6300 Å line occurs about one scale height below the F2 peak (Tuan, 1969) where the electron density oscillation can be larger, (Porter and Tuan, 1974). As an example, for the gravity wave that Thome (1968) analysed, the oscillation in electron density at this height would be between

20% to 25% of the ambient value. If we go down by another say 3/4 of a scale height where the luminosity is still very considerable, the oscillation in electron density rises to about 45% to 50% of its ambient value, thus invalidating any perturbation approach.

An analytic model for the electron density $N_3\ (z,t)$ under the influence of a gravity wave has been developed (Porter and Tuan, loc. cit.) which seems appropriate for analyzing airglow, and is valid for large disturbances. The results are fully time dependent and may be used to study the general decay (including changes in shape) of both the F-layer and airglow in the absence of production of electrons. Finally, we can use this model to study the time variation of the height of the F2 and luminosity peaks which would be difficult to do in a partially time dependent theory that does not allow for any changes in shape in the ambient ionosphere. In this model we have neglected time variations in the temperature, diffusion, and recombination coefficients, (all of which can be incorporated without too much difficulty) since it can be shown that at least for long period gravity waves (Klostermeyer, 1972b, Porter and Tuan, 1974) the percentage variation of these parameters with time are small over most parts of the F-layer. For the case of long period horizontally ducted gravity waves, the expression for the electron density $N_e(z,t)$, (equation (7), Porter and Tuan, loc. cit.) which fully includes effects due to diffusion and recombination is given by

$$N_e = c \exp\left\{-\left[\frac{z}{2H} + \frac{\alpha}{2} e^{-\frac{z}{H}}\right]\right\} \left\{ e^{\left[-\lambda_o (t-t_o) + B\ (q) \sin\omega(t-\bar{t}_o)\right]} - \sum_{n=1}^{\infty} \left[P_n(p,\omega,t)\ \Gamma_n\ (p) + \frac{\Lambda\alpha^q \sin\theta\cos\theta}{H} R_n\ (p,\omega,t)\ G_n(q) \right] \times L_n^{-1/2} (\alpha e^{-\frac{z}{H}}) \right\} \qquad (2)$$

where $z = h-h_o$ is the difference between the actual height h and some arbitrary reference height h_o; H is the scale height; p is the scale height factor; $\lambda_o=\lambda_o(p)$ is a decay constant (see Appendix); ω is the angular frequency of the gravity wave; q is a parameter ($0 \leq q <1/2$) which controls the damping of the gravity wave (when $q = 1/2$ we obtain the undamped Hines Model (Hines, 1960); B(q) is a coefficient depending on gravity wave and ionospheric parameters (see Appendix); $\bar{t}_o$ is some initial time giving us the phase of the gravity wave at $t = t_o$; $\alpha = 2H \sqrt{\frac{\beta_o}{D_o}}$ where β_o and D_o are the recombination and diffusion coefficients at h_o; θ is the angle between the magnetic field and the zenith; A is the amplitude of the gravity wave at h_o; $P_n(p,\omega,t)$ and $R_n(p,\omega,t)$ are oscillating functions of time defined in the appendix $F_n(p)$ and $G_n(q)$ are factors which control the dependence on the scale-height factor and the damping of the gravity wave respectively and are given in the appendix; $L_n^{-1/2}$ is a Laguerre Polynomial of half integral order.

We see at once from equation (2) that if we ignore the contribution from the infinite sum on the right hand side, we would have a Chapman function multiplied by an exponential damping together with oscillations. This is as would be expected since for the night-time situation considered here production can be excluded. (We may easily include production if required).

The infinite sum on the right hand side of equation (2) reduces to a few terms for a gas either perfectly mixed or in diffusive equilibrium and for a constant velocity amplitude (independent of height) for the gravity wave ($q = 0$), (see Appendix).

In the case when there is no gravity wave ($A = 0$) or when we are at the geomagnetic pole ($\Theta = 0$) or equator ($\Theta = \frac{\pi}{2}$), $B(q) = 0$ (see Appendix) and equation (1) reduces to the expression for an undisturbed ionosphere (Moschandreas and Tuan, 1973). The decay constant $\lambda_o(p)$ describes the general F-layer decay in the absence of disturbance and was originally derived through perturbation theory (Tuan, 1968). In general $\lambda_o(p)$ increases when the gases are in diffusive equilibrium ($p = 2$).

The expression given by equation (2) provides the correct phase behavior as well as the F2 peak motion, as is seen from a comparison with Thome's (1969) observations (Porter and Tuan, 1973). By inserting the above expression for $N_e(z,t)$ in equation (1) we obtain the luminosity profiles for the 6300 Å OI shown in Fig. 1. Here we have adopted similar ionospheric and gravity wave parameters as those used to fit Thome's (1968) observations except for the scale-height factor for which we have now used $p = 1.75$ in order to bring the gases closer to diffusive equilibrium as one might expect for the F-region. We shall assume that $A_{1_D} = 1/110$ sec^{-1} (Chamberlain, 1961), $S_1 = 5 \times 10^{-11}$ cm^3 sec^{-1} (Peterson and Van Zandt, 1969) and $S_2 = 1.7 \times 10^{-9}$ cm^3 sec^{-1} (Seaton, 1956).

In Fig. 1, a comparison of the electron density profile during the passage of a gravity wave with the movements of the luminosity profile seems to show that (1) in general the luminosity profile peaks about one scale height below the F2 peak as in the undisturbed time-dependent theory (Tuan, 1969), (2) one hour later, as the F2 peak moves down by 20 Km losing all the time in electron density magnitude, the luminosity peak remains relatively unchanged in position, but greatly enhanced in intensity (Fig. 1 (b)), (3) another hour later, as the F2 peak moves up again, the intensity of peak luminosity again drops with little rise in

position (Fig. 1(c)). If we had plotted the luminosity profiles without quenching, the peak luminosity would have moved a considerable distance (about 15 Km) downward when the F2 peak comes down. This implies that the luminosity peak is essentially kept up by the rapid increase in quenching at lower altitudes. The reason that it would only move 15 km rather than the 20 km for the F2 peak is due to the rapid drop in available O_2^+ for the production of ^1D states at higher altitudes which tends to keep the luminosity peak down. Thus, the combined effect of both is to keep the luminosity peak from having larger oscillations in the vertical direction.

Oscillations in total electron content and 6300 Å OI

To obtain the total zenith airglow intensity for the 6300 Å OI we integrate equation (1) over height. The results are plotted against local time in Fig. 2. We have also plotted the total columnar electron content as well as the negative time derivative of this total electron content. The parameters used here are the same as those adopted for Fig. 1. One general feature is that all three curves decay with time, an expected result since no ion-pairs production mechanism have been included. The most significant feature is that while there is considerable oscillation with time in the airglow intensity, the total electron content shows relatively little fluctuation. The lack of fluctuation in the total electron content is readily understandable. We are first of all dealing with a night-time situation when we may neglect photoproduction. The fluctuations arise when the ions are moved alternately into rarified atmosphere where the recombination is less and denser atmosphere where the recombination is greater. The fluctuation is further inhibited by phase cancellation between the motion of ions above and below the F2 peak. Since we are dealing with a horizontally ducted neutral wave with no vertical phase progression, the phase variation of ions depends almost entirely on the shape of the F-layer. Fig. 6 shows how the phase of the ion motion above the vicinity of the F2 peak is in general opposite to that below the peak. We have here an example of possible phase cancellation even when the gravity wave does not possess phase variation along the line of sight. Georges and Hooke (1970) have shown how phase cancellation may occur from phase variations in the gravity wave itself.

The columnar airglow intensity, on the other hand, is proportional to the ion-pair recombination if we neglect quenching. By integrating the continuity equation for electrons over z and dropping the production term we obtain the result that the columnar recombination rate is equal to the negative of the rate of change of columnar electron content since the divergence term in the continuity equation drops out from Gauss Theorem. Thus, if the quenching were neglected, we would expect the airglow intensity to be directly proportional to the negative of the rate of change of total electron density. This is shown very clearly in Fig. 2 in which the airglow fluctuations follow the negative of the rate of change of total electron content closely. Both curves are opposite in phase to the total electron content as expected. If the quenching terms are very large, then a phase difference will be introduced between the airglow fluctuations and the rate of change of total electron content. In general, there are two possible ways to introduce this phase difference. One is to introduce quenching as we have just mentioned. The other is to introduce some production process. This point will be further discussed later.

The 5200 Å Nitrogen Line

The 5200 Å NI is produced in the F-region by dissociative recombination in a manner similar to the 6300 Å line. Analysis of the effects of a gravity wave on this line is of special interest because of the very long lifetime of the nitrogen $^2D_{5/2}$ states. We can no longer use the quasi-equilibrium theories, especially at higher altitudes where quenching can no longer compensate for the very slow radiative loss. If we neglect diffusion (Hernandez and Turtle, 1969), the continuity equation for $N(^2D)$, the atomic nitrogen density in the ^2D state is given by:

$$\frac{\partial N(^2D)}{\partial t} = P - (A_{^2D} + K_1 \left[O_2\right] + K_2 N_e) \quad N(^2D) \tag{3}$$

In equation (3) P is the production rate for $N(^2D)$, K_1 and K_2 are the quenching coefficients for O_2 and electrons respectively, $A_{^2D}$ is the Einstein coefficient for $N(^2D)$. If we go down to low enough altitudes where the quenching is sufficiently strong to balance the production rate, we may neglect the time derivative term and the theory reduces to a quasi-equilibrium theory exactly analogous to that of Peterson, et.al. (1966). For the production rate we again assume that the production of $N(^2D)$ through dissociative recombination of NO^+ is balanced by charge transfer between O^+ and N_2. In this way the production rate of $N(^2D)$ states varies with height as the product of neutral N_2 concentration and N_e.

The loss rate of $N(^2D)$ is represented by the three terms on the right-hand side of equation (3), corresponding to radiative loss, O_2 quenching and quenching due to superelastic collisions with the electrons. We have assumed that $A_{^2D} = 1/26$ hr (Chamberlain, 1961) and $K_2 = 8 \times 10^{-10}$ cm^3 sec^{-1} (Seaton, 1956). For the rate

coefficient K_1 with respect to O_2 quenching we have tried both $K_1 = 1.4 \times 10^{-11}$ $cm^3\ sec^{-1}$ (Slanger, et. al., 1971) and $K_1 = 1.4 \times 10^{-12}$ $cm^3\ sec^{-1}$ (Hernandez and Turtle, 1969). The two values differ by an order of magnitude and there is reason to believe that the value given by Hernandez and Turtle may be on the small side (Hernandez, private communication). Equation (3) may be simply integrated subject to the initial boundary conditions. From the $N(^2D)$ concentration we may readily calculate the columnar zenith airglow intensity for the $(^4S_{3/2} - {}^2D_{5/2,3/2})$ forbidden optical transition as a function of time. The results are shown by the solid curve in Fig. 3 for the quenching coefficient of Slanger, et. al. (1971) and the dotted curve for the coefficient used by Hernandez and Turtle (1969). For both curves we have used a scale-height factor of $p = 1.75$. All other gravity wave parameters are again the same as those used to fit Thome's results, (Porter and Tuan, loc, cit.).

The three important features are (1) the oscillations for the higher O_2 quenching rate of Slanger, et. al. (1971) is less pronounced than those produced by the lower quenching coefficient of Hernandez and Turtle (1969), (2) there is a phase advance with the higher quenching coefficient of between 10 to 20 minutes. The reasons for this will be given in a later section, (3) the luminosity peak for the higher quenching coefficient is in general about 20 km higher than the peak for the lower coefficient.

In comparing these results with the behavior of the 6300 Å line, there are the following important differences: The electron quenching for the nitrogen line is relatively speaking much more important than for the oxygen line. If we use the coefficient of Slanger, et.al. (1971), the quenching due to molecules

for the nitrogen line is down by more than an order of magnitude from the quenching for oxygen due to the combined effect of a smaller quenching coefficient as well as a smaller abundance of O_2 molecules in comparison with N_2 molecules. The difference in electron quenching is only about a factor of 2.

The relatively more important electron quenching has the effect of holding down the 5200 Å luminosity peak, since the available electrons for quenching increase until the F2 peak, well above the luminosity peaks of both the 6300 Å and the 5200 Å lines.

The net result is that although one may at first expect the 5200 Å luminosity peak to be well above the oxygen peak, owing to a much longer half-life, the reduced molecular quenching at lower altitudes coupled with a relative increase in electron quenching (unimportant for the 6300 Å line) at higher altitudes would bring down the peak position for the 5200 Å line to comparable height levels as that for the 6300 Å line. This is not inconsistent with the observations of Weill and Christophe-Glaume (1967) who argued from geometric considerations that the 5200 Å peak is about 12 km above the 6300 Å peak if the speed of the assumed TID is assumed to be the same at the peaks of both emissions. We may also at the same time conclude that since both peaks occur at nearly the same height, the oscillation amplitudes should be comparable. This is borne out by comparing Fig. 3 with Fig. 2.

Comparison with simultaneous observations of foF2, h'F2 and 6300 Å OI emission

We have mentioned earlier that some published data seems likely to have exhibited the effects of gravity waves. In this section we use some of Dach's (1968) simultaneous measurements of ionospheric parameters and the 6300 Å OI emission to illustrate the theory by fitting the observations to an assumed gravity wave.

Dachs (1968) has made intensity measurements of the 6300 Å and the 5577 Å emission lines as a function of time. These measurements together with simultaneous measurements of foF2 and h'F2 seem to show considerable correlation, especially between the 6300 Å line and the ionospheric parameters. In Fig. 4 we have shown a comparison between our theoretical results and the experimental values for foF2, h'F2 and the 6300 Å airglow. For the theoretical parameters we have chosen $p = 1.5$, $H = 65$ Km, the recombination coefficients $\beta(200 \text{ Km}) = 0.34 \text{ hr}^{-1}$ and the diffusion coefficient $D(200 \text{ Km}) = 6.1 \times 10^3 \text{ Km}^2 \text{ hr}^{-1}$. These are somewhat different from the parameters chosen to fit Thome's observation but considering the fact that we have completely ignored production processes (the observations are made between 1800 and 2400 L.T.) they are the right order of magnitude. For the gravity wave, we assume the velocity amplitude $A(200 \text{ Km}) = 35.2 \text{ m sec}^{-1}$ and the damping factor $q = 0.49$ which is rather close to the Hines (1960) undamped model. The quenching at 200 Km is assumed to be 0.24 sec^{-1} and the electron quenching is neglected. The quantum efficiency for emission from the ^1D is assumed to be 0.5.

In view of the rather good agreement between theory and experiment for both the airglow and the ionospheric parameters, one can readily conclude that even without a total electron content measurement we have a gravity wave passing

through with a period of between 2 to 3 hours. The comparison between the airglow intensity and the h'F2 show all the expected phase correlations, i.e. when the airglow intensity is at a minimum the F2 peak is approximately at a maximum.

<u>Effect of quenching and the criteria for different disturbances</u>

As already mentioned in a previous section, quenching can cause a phase difference in the oscillation of the airglow intensity. Fig. 5 shows the 6300 Å OI for two N_2 quenching coefficients. The solid curve uses a quenching coefficient $S_1 = 1.0 \times 10^{-11}$ cm^3 sec^{-1} while the dashed curve uses $S_1 = 2.7 \times 10^{-11}$ cm^3 sec^{-1}. All other parameters are the same as those used to fit Dach's (1969) experimental observations. The higher quenching coefficient seems to produce less oscillations as well as a phase advance by about half an hour. It is easy to understand why the oscillation amplitude should decrease when the quenching is increased since a higher quenching rate would move the luminosity peak up to where the electron density oscillation is less. This is true so long as the peak luminosity remains below the F2 peak, (Porter and Tuan, 1973).

To understand why we should have a phase advance we must consider the variation in the phase of the electron density oscillation with respect to the neutral gravity wave as a function of height. The solid curve in Fig. 6 shows a plot of the theoretical phase variation with altitude while the dashed curve shows Thome's (1968) observed results, (Fig. 6 is taken from Porter and Tuan, 1973). We see that the phase of the electron density oscillations advances monotonically with height. There is a region, however, below 260 Km where an increase in the height of the luminosity will bring little phase change. Above 260 Km, any increase in the height of the luminosity curve would bring a rapid advance in the phase of the electron density oscillation. Even though Fig. 5 deals with an integrated intensity, since the phase advance is monotonic, an upward shift of the entire luminosity profile must always produce a phase advance.

The analysis seems to suggest a means for distinguishing airglow fluctuations

caused by gravity waves from those caused by incident particle flux, if a simultaneous measurement of total electron content and airglow intensity can be made. If no production mechanism is present and the disturbance is caused entirely by a gravity wave, then (1) there is relatively little fluctuation in the total electron content or in any experimental parameter that depends only on the total number of columnar electrons rather than, say, how the electrons are distributed; (2) the rate of change of total electron content should remain negative. This means that for night-time conditions and a gravity wave length (of the order of 1500 Km say) long compared with the thickness of the F-layer so that we may neglect horizontal transport arising from concentration gradients in the horizontal direction, the total columnar electron content should decrease monotonically with time. This is easily understandable since the only reason for the little fluctuations that we do get in the total electron content is due to the variation in the recombination coefficients as the ionosphere moves up and down; (3) there is always a phase advance in the airglow fluctuations with respect to the rate of change of total electron content. The magnitude of the advance is in general determined by the quenching. We should remark here that if a horizontal time varying electrostatic field exists in the F-region causing an $\vec{E} \times \vec{B}$ drift in the ambient electrons only without any horizontal transport then all three of the above properties in the behavior of the airglow and columnar electron density should also hold. They would no longer be true, however, if somehow additional electrons are introduced either through transport or some sort of production mechanism. For instance, if the disturbance is caused by an incident particle flux which produces additional

ion-electron pairs, then the rate of change of total electron content can go positive since the total columnar electron content can no longer vary monotonically with time. In general, the fluctuation can also be far more drastic (Moschandreas and Tuan, 1973).

As an example, we consider the data of Brown and Steiger (1972) who have simultaneously measured the 6300 Å airglow and the total electron content as a function of time at Hawaii. They found an oscillation in the airglow which correlates with oscillations in the total electron content. The negative time derivative of columnar electron content trails the airglow oscillation by a time delay of about 25 minutes. We would have at first some reason to feel that this particular disturbance could be caused by a wave since (1) the disturbance appears to be periodic with a period of about 3 1/2 hours; (2) the airglow fluctuation shows a phase advance of about 20 to 30 minutes with respect to the rate of change of total electron content. However, it is rather unlikely that the disturbance is a wave since the period is of the order of 3 1/2 hours and the disturbance takes place around the middle of the night when we can forget about photoionization. The disturbance also does not vary monotonically with time and shows large oscillation amplitudes. We may add that condition (1) mentioned above can be produced by any periodic disturbance and (2) can be produced by either a gravity wave or a vertical drift without any introduction of additional charged particles. There is, of course, a possibility that we may simultaneously have an electromagnetic drift together with an introduction of additional electrons. In fact, such an event would be quite consistent with both the experimental observations as well as our criteria.

Conclusion

We have examined the effect of a gravity wave in the F-region on the electron density and on the emission intensity of two forbidden transitions whose mechanisms involve recombination with electrons. Because of such effects as quenching and the lack of available O_2^+ and NO^+ molecules at high altitudes, the verticle movement of the luminosity profiles in response to the passage of the gravity wave is much smaller than that of the electron density profile. The amplitude of oscillation of the airglow, on the other hand, is greater than that of the total electron density which varies monotonically with time. Owing to the presence of a magnetic field as well as the shape of the electron density profile, phase differences in behavior occur between different heights. This effect, combined with the height differentiating effect of quenching produces phase differences also between airglow and ionospheric parameters. A combination of airglow and ionospheric measurements can be used to differentiate between gravity waves and the effects of incident particle flux.

The theory has been tested by assuming a gravity wave effect for some published data on simultaneous measurements of airglow and ionospheric parameters. The fit was satisfactory.

This study represents the first theoretical treatment, as far as we are aware, of the effects of gravity waves on airglow. We have tried here to establish the basic characteristics to be expected for the luminosity and electron density profiles under the influence of a gravity wave, and to show how these could be used in some cases to provide information on processes. Expansion of the techniques should be useful as a tool for the study of both gravity waves and upper atmosphere parameters and processes.

Appendix

A good analytical approximation for the decay constant λ_o expressed as a function of the scale height factor p is given by:

$$\lambda_o(p) = \frac{\sqrt{B_o D_o}}{2\ H} \left[1 + \frac{1}{\pi^{\frac{1}{2}}} \left(\alpha^{1-p}\ \gamma(p + \tfrac{1}{2}) - \gamma(1 + \tfrac{1}{2}) \right) \right] \qquad \text{(A 1)}$$

A derivation for this is given by Tuan (1968). Another totally different derivation has been given by Porter and Tuan (1974). In this expression λ_o decreases monotonically with increasing p, where $1 \leq P \leq 2$. Thus, the decay of the F-layer is in general faster when the gases are perfectly mixed (P = 1).

The expression B(q) in equation (2) is given by:

$$B(q) = \frac{A\alpha^q \sin\theta \cos\theta\ \gamma\ (\frac{3}{2} - q)}{2\pi^{\frac{1}{2}}\ \omega H} \qquad \text{(A 2)}$$

As already mentioned, q varies between 0 and 1/2. If q is set to zero the velocity amplitude does not vary with height. Otherwise, it increases exponentially. We see that B(q) vanishes at the magnetic pole and equator where we do not expect a horizontally ducted gravity wave to produce oscillations in airglow.

The functions $P_n(p,\omega,t)$ and $R_n(p,\omega,t)$ are defined (Porter and Tuan, 1974) by:

$$P_n = c(q,t_o,\bar{t}_o) \int_{t_o}^{t} e^{\frac{\omega_n}{k}(t-t')}\ e^{-\left[\lambda_o(t'-t_o)-B(q)\sin\omega(t'-\bar{t}_o)\right]} dt' \qquad \text{(A 3)}$$

$$R_n = c(q,t_o,\bar{t}_o) \int_{t_o}^{t} e^{\frac{\omega_n}{k}(t-t')}\ e^{-\left[\lambda_o(t'-t_o)-B(q)\sin\omega(t'-\bar{t}_o)\right]} \cos\omega(t' - \bar{t}_o)\ dt' \qquad \text{(A 4)}$$

where
$$\omega_n = -(n + 1/4)$$
$$k = \frac{H}{2\sqrt{B_o D_o}}$$

While both P_n and R_n are oscillating functions of time with frequency R_n may change sign but P_n remains always positive. It is easy to see $\omega \to 0$, both P_n and R_n become monotonic functions of time and all oscil vanish from the expression given by equation (2).

The functions $F_n(p)$ and $G_n(q)$ are given by:

$$F_n(p) = \frac{\alpha^{1-p}\Gamma(n-p)\,\Gamma(n+\frac{1}{2})}{\Gamma(-p)} - \frac{\Gamma(n-1)\,\Gamma(1+\frac{1}{2})}{\Gamma(-1)} \quad \text{(A}$$

$$G_n(q) = \frac{\Gamma(q+n-1)\,\Gamma(^3/_2 - q)}{\Gamma(q)}\left[\frac{(1-q)}{2} - n\right] \quad \text{(A}$$

The derivation of these functions are given in Tuan (1958) and Por and Tuan (1974). They have the following important property:

$$F_n(p = 1) = 0 \text{ for all } n$$
$$F_n(p = 2) = 0 \text{ for all } n \geq 3$$
$$G_n(q = 0) = 0 \text{ for all } n \geq 2$$

Hence, the infinite sum on the right-hand side of equation (2) wil to only a few terms for a gas either perfectly mixed or in diffusive e and for a constant velocity amplitude ($q = 0$).

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Figure Legends

Fig. 1. The comparison of the motion of the electron density profile with 6300 Å luminosity profile under the influence of a gravity wave.

Fig. 2. Time variation of the 6300 Å zenith airglow is compared with the total columnar electron content and the time derivative of the total electron content.

Fig. 3. Time variation of the 5200 Å nitrogen emission in a gravity wave for the O_2 quenching coefficient of Hernandez and Turtle (dashed curve) and Slanger et.al. (solid curve).

Fig. 4. The experimental observations of Dachs showing simultaneous variations in foF2, h'F2 and the 6300 Å airglow are compared with theoretical calculations (solid curves) based on the assumption of a gravity wave with a period of approximately 3 hours.

Fig. 5. The oscillations of the 6300 Å line for two nitrogen quenching coefficients, showing how the larger quenching coefficient (dashed curve) reduces oscillation and introduces a phase advance.

Fig. 6. Comparison of the theoretical calculations of the variation with altitude in the phase of the ionospheric oscillations with Thome's (1968) observed values.

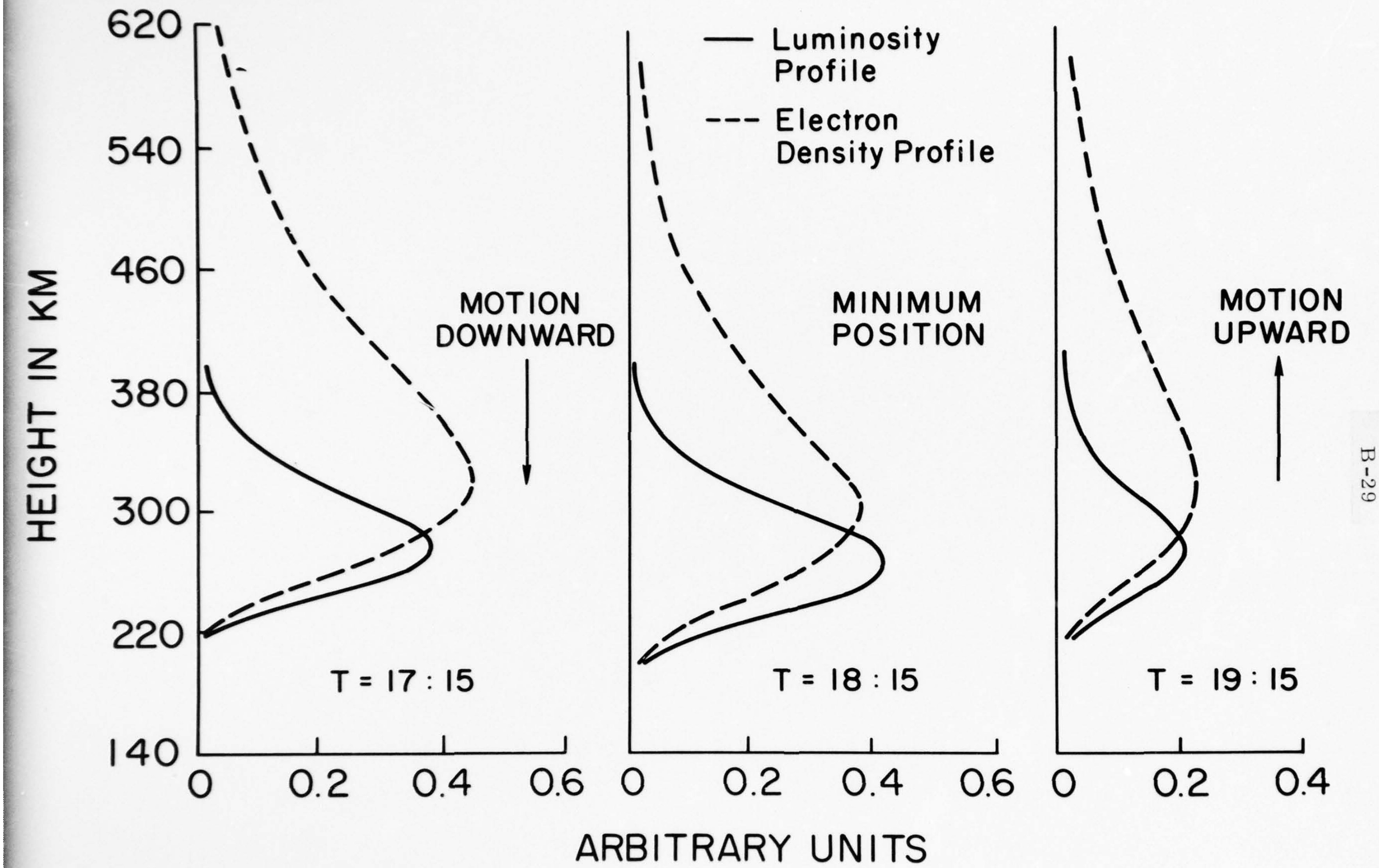
HEIGHT IN KM
620
540
460
380
300
220
140
Luminosity Profile
Electron Density Profile
MOTION DOWNWARD
MINIMUM POSITION
MOTION UPWARD
T = 17 : 15
T = 18 : 15
T = 19 : 15
0 0.2 0.4 0.6
0 0.2 0.4 0.6
0 0.2 0.4
ARBITRARY UNITS

FIG. 1

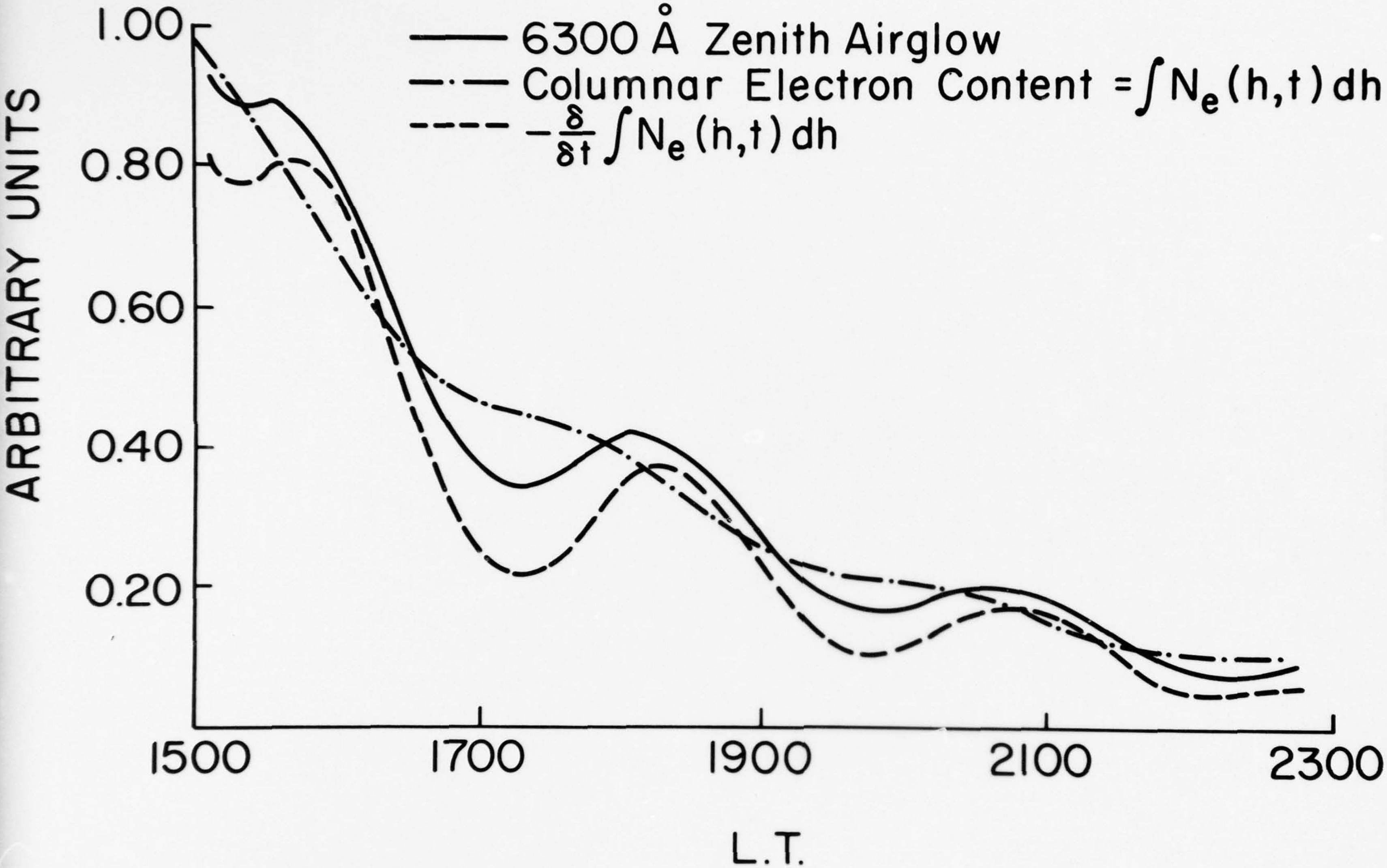
6300 Å Zenith Airglow
Columnar Electron Content = $\int N_e(h,t)\,dh$
$-\frac{\delta}{\delta t}\int N_e(h,t)\,dh$
ARBITRARY UNITS
1.00
0.80
0.60
0.40
0.20
1500
1700
1900
2100
2300
L.T.

FIG. 2

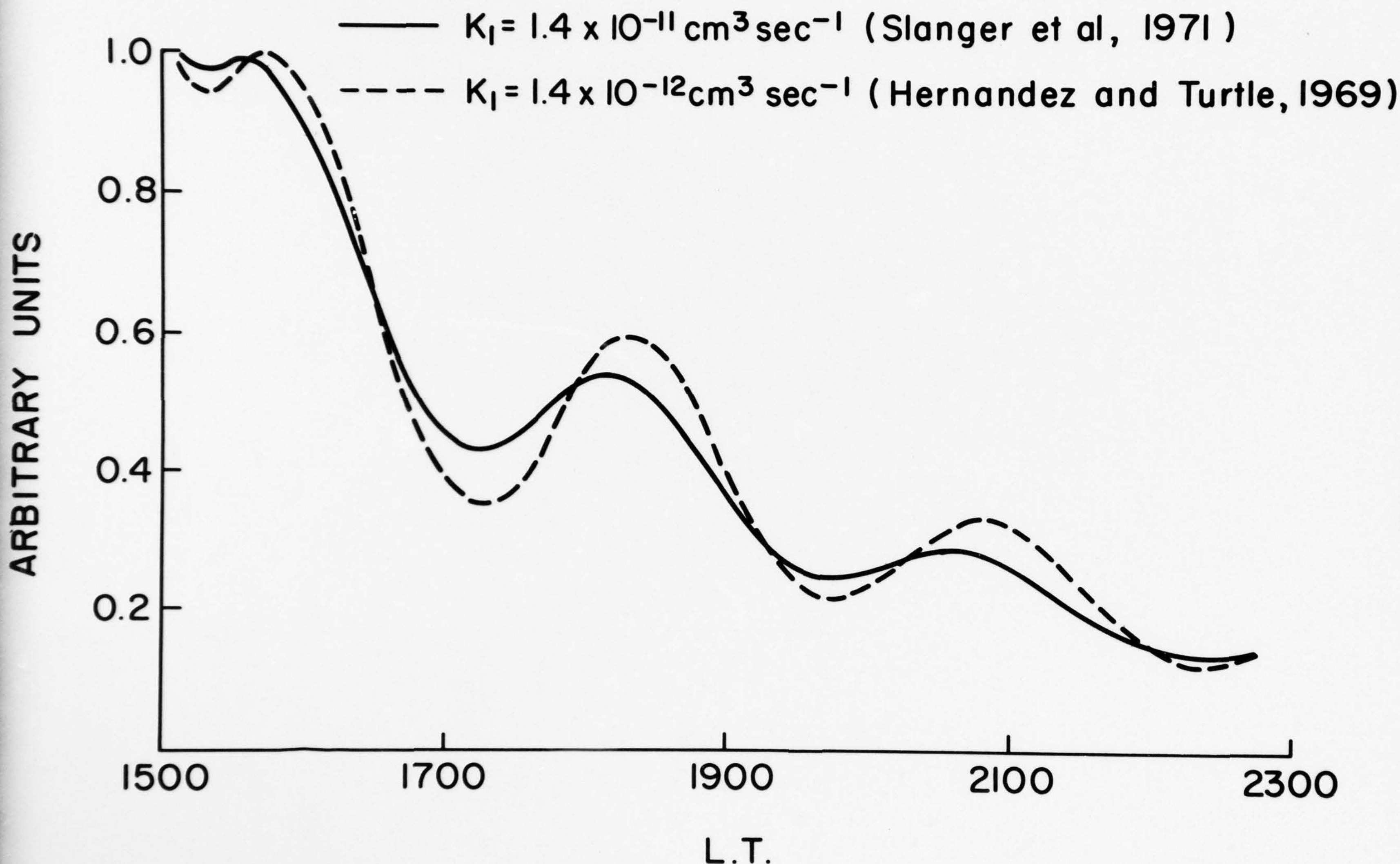
$K_1 = 1.4 \times 10^{-11}\ cm^3\ sec^{-1}$ (Slanger et al, 1971)
$K_1 = 1.4 \times 10^{-12}\ cm^3\ sec^{-1}$ (Hernandez and Turtle, 1969)
1.0
0.8
0.6
0.4
0.2
ARBITRARY UNITS
1500
1700
1900
2100
2300
L.T.

FIG. 3

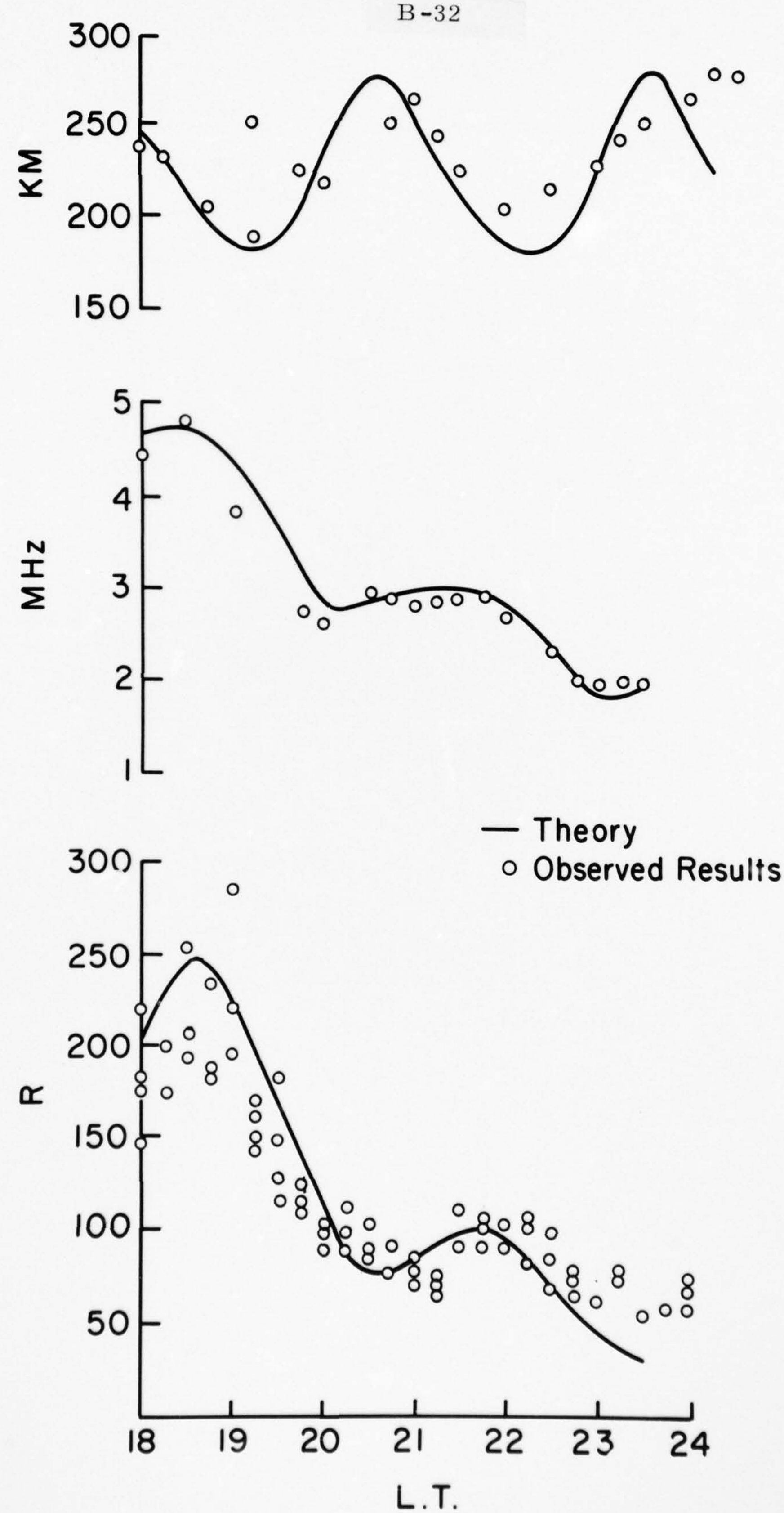
300
250
200
150
KM
5
4
3
2
1
MHz
Theory
Observed Results
300
250
200
150
100
50
R
18
19
20
21
22
23
24
L.T.

FIG. 4

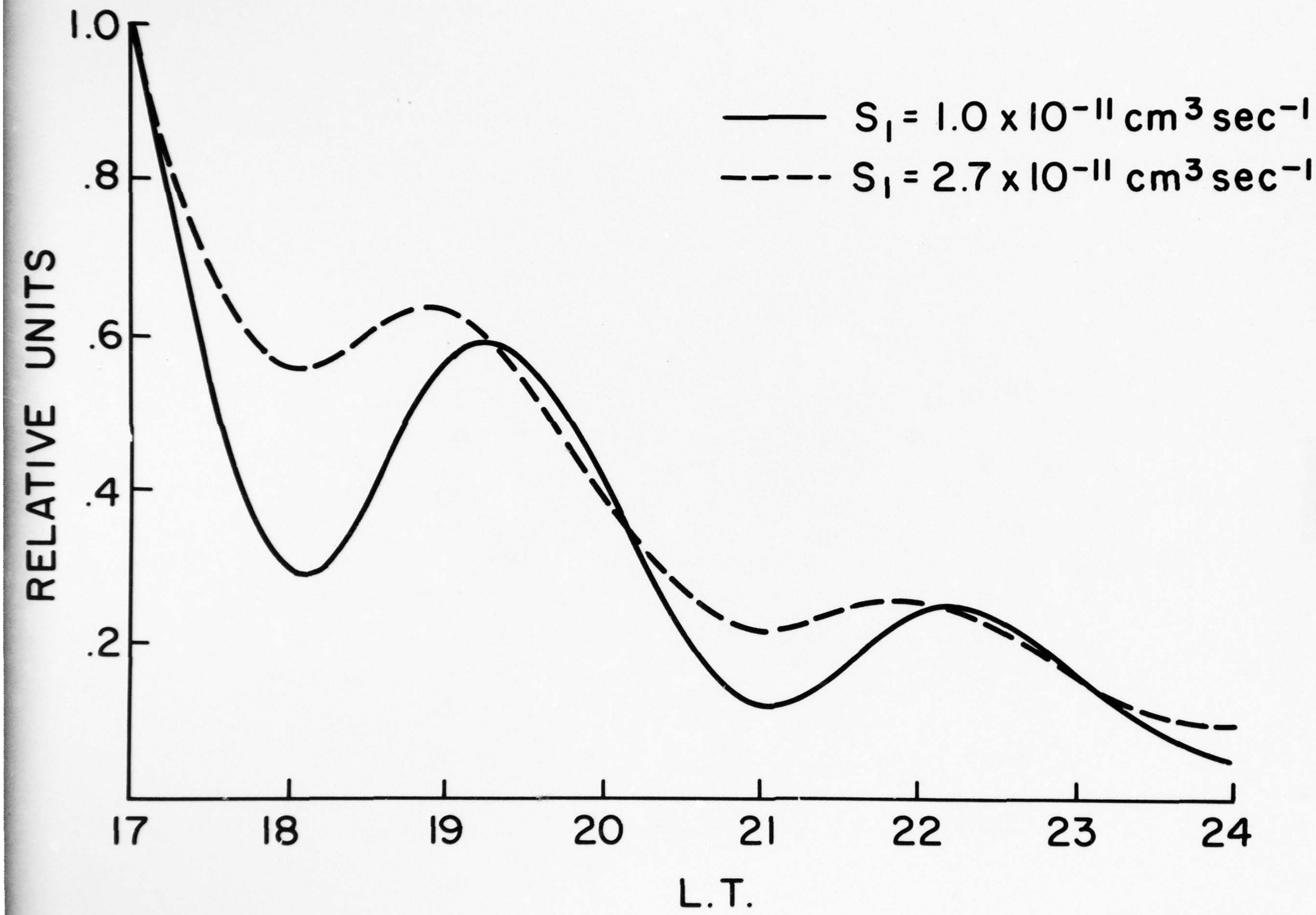
RELATIVE UNITS
1.0
.8
.6
.4
.2
17
18
19
20
21
22
23
24
L.T.
$S_1 = 1.0 \times 10^{-11}\ cm^3\ sec^{-1}$
$S_1 = 2.7 \times 10^{-11}\ cm^3\ sec^{-1}$

FIG. 5

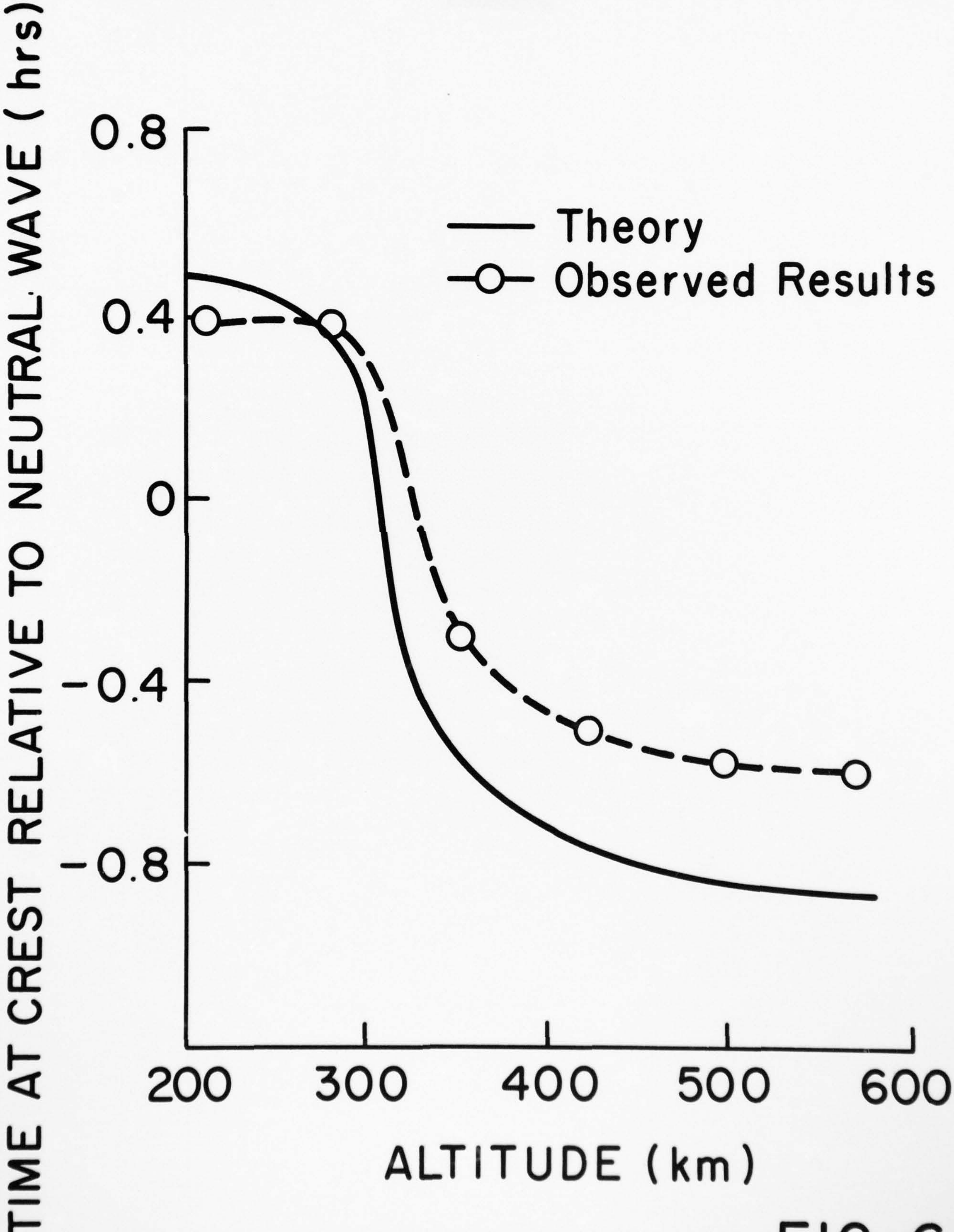
TIME AT CREST RELATIVE TO NEUTRAL WAVE (hrs)
0.8
0.4
0
-0.4
-0.8
Theory
Observed Results
200
300
400
500
600
ALTITUDE (km)

FIG. 6

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On the Behavior of Airglow Under the Influence of Gravity Waves

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Although the experimental observations of airglow intensity fluctuations and their possible connections with traveling ionospheric disturbances have been made by many investigators, very little attention has been given to a theoretical analysis of such ionospheric disturbances on airglow. In this paper we specifically examine the effect of an atmospheric gravity wave on the 6300-Å O I and the 5200-Å N I emission in mid-latitudes. We will also show that for the 6300-Å O I if we assume reasonable values for the ionospheric parameters as well as a gravity wave with a period of approximately 3 h, the resultant fluctuations in airglow are consistent with Dachs's observed data as well as with the simultaneously measured variations in the ionospheric parameters.

An association of airglow intensity variations with traveling ionospheric disturbances has been established by *Barbier* [1964, 1965] for the 6300-Å O I emission and by *Weill and Christophe-Glaume* [1967] for both the 6300-Å O I and the 5200-Å N I emission by a direct comparison of the two types of measurements. In addition, some published data [*Silverman*, 1962; *Okuda*, 1962; *Dachs*, 1968] can in retrospect be seen as having been probably due to similar causes. The data of Dachs include observations on both 5577-Å O I and 6300-Å O I as well as observations of the ionospheric parameters f_0F_2 and $h'F_2$ and are thus particularly valuable for analysis from the point of view of a possible wave origin of the variations. Disturbances in airglow can of course arise from a variety of causes, and simultaneous measurements of other parameters are needed to establish the cause of a disturbance.

Over the past few years considerable theoretical and experimental work has been carried out on the effects of atmospheric gravity waves on ionospheric parameters. The effects of these waves on optical emissions, however, have received no attention from a theoretical viewpoint. A suitable analytic model for such a treatment has been presented recently by *Porter and Tuan* [1974]. This model for the response of the F layer to gravity waves differs fundamentally from the usual theoretical treatment in several respects. Of these the most pertinent for airglow applications are the following:

1. The analytic model is fully time dependent. One result of such a model is that given a sinusoidal neutral gravity wave, the response of the electron density oscillation is no longer in general sinusoidal and the time behavior varies with height. This makes it more suitable for studying both the general decay of the F layer and the airglow at night as well as such detailed behavior as differences in phase between the oscillations of electron density and the airglow.

2. The analytic model remains valid even if the percentage oscillations in the electron density or airglow are large in comparison with ambient values. This condition is especially important for the 6300-Å O I emission, whose luminosity profile peaks at a position (about one scale height below the F_2 peak) where the electron density and airglow oscillations are rather large.

In this paper we first outline the computation for the 6300-Å O I emission intensity during the passage of a gravity wave. For long-period gravity waves the continuity equation from which the emission intensity of the 6300-Å O I can be calculated reduces to the quasi-equilibrium theories of *Peterson et al.* [1966], in which the production of the excited $O(^1D)$ states is assumed to equal the loss through optical emission and quenching. Comparison of the theoretically calculated changes in both electron density and luminosity profiles will serve to bring out the essential features of the theory.

We shall next consider the time behavior of the total electron content as compared with the airglow oscillations. It will be shown that the total electron content shows less oscillatory behavior and that only by plotting the time derivative of the total electron content can the oscillations be more significant. However, such fluctuations are detectable [*Davis and daRosa*, 1969]. The reduced oscillation in the total electron content, coupled with the large optical emission oscillations, provides us with a possible criterion for distinguishing between long-period gravity wave effects and effects produced by other sources such as particle participation.

We will then consider the effect of a gravity wave on the 5200-Å N I emission intensity. Here we can no longer use the equilibrium theory that we have been using for the 6300-Å line, since at least at higher altitudes the production of $N(^2D)$ states can no longer be balanced by either the radiative loss or the quenching. We will also consider the importance of elec-

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tron quenching, which is not significant for the 6300-Å O I line, but, as we shall see, is much more important for the nitrogen line.

The theory derived here will then be used for the interpretation of the experimental results of *Dachs* [1968] at Tsumeb, South-West Africa. These include simultaneous measurements of f_oF_2, $h'F_2$, and the airglow intensity variations with time, which allow for a test of both aspects of the theory.

Finally, we shall also study the effect of varying the quenching rates on the phase of the oscillations. We will consider this phase difference in relation to the effect of other mechanisms that can also cause a phase difference.

TIME BEHAVIOR OF ELECTRON DENSITY AND 6200-Å O I LUMINOSITY PROFILES

Theoretical treatments of the 6300-Å O I emission taking account of different factors and calculating various parameters are available from a number of authors [e.g., *Chamberlain*, 1958; *Lagos et al.*, 1963; *Lagos*, 1964; *Peterson et al.*, 1966; *Tuan*, 1969]. The following treatment leans most heavily on the work of Peterson et al.

We make the following assumptions (for their justification see *Peterson et al.* [1966]): (1) the time derivative term in the continuity equation for the density of oxygen atoms in their $O(^1D)$ states may be neglected, (2) the diffusion term is neglected, and (3) we assume chemical equilibrium for the production and loss of O_2^+ molecules and neglect the O_2^+ and NO^+ molecules in the equation for charge neutrality. We then obtain the following well-known expression for the volume emission rate ϵ_{6300} for the 6300-Å line:

$$\epsilon_{6300}(z, t) = \eta\left(\frac{A_{6300}}{A_{^1D}}\right)\frac{\gamma[O_2]N_e}{1 + (1/A_{^1D})(S_1[N_2] + S_2N_e)} \tag{1}$$

where η is the efficiency for recombination, γ is the rate coefficient for charge transfer between O and O_2^+, and S_1 and S_2 are the nitrogen and electron quenching coefficients, respectively. The O_2^+ profiles obtained by *Schunk and Walker* [1973] could later be used in (1). However, since the same chemical equilibrium and charge neutrality approximations have been used in the expression for N_e [*Porter and Tuan*, 1974], we have decided to maintain consistency and use (1) for the emission rate. Since our primary interest is in the basic dynamic response of the airglow to gravity waves rather than any detailed behavior, we have also neglected the contribution from dissociative recombination of NO^+ [*Silverman*, 1970] as well as the cascading from the ^1S states in the production of ^1D states [*Rishbeth and Garriott*, 1969]. We will also neglect the time variation in O_2 and N_2 caused by the gravity waves. In reality, the equations for such neutral atmospheric constituents are really coupled to the equation for the ion-electron plasma. To simplify all these processes, we have used stationary models for O_2 and N_2. In practice the approximations should be good, since, as was already mentioned, the percentage variation in the neutral gas caused by the gravity wave is usually very much less than the percentage variation for the ion-electron plasma, especially in important regions such as the peak of the luminosity profile. By far the most important contribution to the oscillation in the airglow arises from changes in the electron density N_e in (1).

Most recent theoretical treatments for the time-dependent behavior of F layer electron density [e.g., *Testud and Francois*, 1971; *Klostermeyer*, 1972*a*, *b*] under the influence of gravity waves are essentially based on perturbation theory in which the unperturbed ionosphere is assumed to be a stationary Chapman function independent of time. Also, in perturbation treatments the time dependence for the plasma is assumed to be sinusoidal if the gravity wave is sinusoidal, and a simple product form for the time and spatial dependence is usually assumed.

The luminosity peak for the nighttime 6300-Å line occurs about one scale height below the F_2 peak [*Tuan*, 1969], where the electron density oscillation can be larger [*Porter and Tuan*, 1974]. As an example, for the gravity wave that *Thome* [1968] analyzed the oscillation in electron density at this height would be between 20 and 25% of the ambient value. If we go down by another, say, three quarters of a scale height, where the luminosity is still very considerable, the oscillation in electron density rises to about 45–50% of its ambient value; thus any perturbation approach is invalidated.

An analytic model for the electron density $N_e(z, t)$ under the influence of a gravity wave has been developed [*Porter and Tuan*, 1974] that seems appropriate for analyzing airglow and is valid for large disturbances. The results are fully time dependent and may be used to study the general decay (including changes in shape) of both the F layer and the airglow in the absence of production of electrons. Finally, we can use this model to study the time variation of the height of the F_2 and luminosity peaks, a study that would be difficult in a partially time-dependent theory that does not allow for any changes in shape in the ambient ionosphere. In this model we have neglected time variations in the temperature, diffusion, and recombination coefficients (all of which can be incorporated without too much difficulty), since it can be shown that at least for long-period gravity waves [*Klostermeyer*, 1972*b*; *Porter and Tuan*, 1974] the percentage variation of these parameters with time is small over most parts of the F layer. For the case of long-period horizontally ducted gravity waves, the expression for the elctron density $N_e(z, t)$, [*Porter and Tuan*, 1974, equation (7)], which fully includes effects due to diffusion and recombination, is given by

$$\begin{aligned} N_e = {} & c \exp\left\{-\left[\frac{z}{2H} + \frac{\alpha}{2}\exp\left(-\frac{z}{H}\right)\right]\right\} \\ & \cdot\left\{\exp\left[-\lambda_0(t - t_0) + B(q)\sin\omega(t - \bar{t}_0)\right]\right. \\ & - \sum_{n=1}^{\infty}\left[P_n(p, \omega, t)F_n(p)\right. \\ & \left. + \frac{A\alpha^q \sin\theta\cos\theta}{H} R_n(p, \omega, t)G_n(q)\right] \\ & \left. \times L_n^{-1/2}\left[\alpha\exp\left(-\frac{z}{H}\right)\right]\right\} \end{aligned} \tag{2}$$

where $z = h - h_0$ is the difference between the actual height h and some arbitrary reference height h_0; H is the scale height; p is the scale height factor; $\lambda_0 = \lambda_0(p)$ is a decay constant (see appendix); ω is the angular frequency of the gravity wave; q is a parameter ($0 \leq q < \frac{1}{2}$) that controls the damping of the gravity wave (when $q = \frac{1}{2}$, we obtain the undamped *Hines* [1960] model); $B(q)$ is a coefficient depending on gravity wave and ionospheric parameters (see appendix); $\bar{t}_0$ is some initial time giving us the phase of the gravity wave at $t = t_0$; $\alpha = 2H(\beta_0/D_0)^{1/2}$, where β_0 and D_0 are the recombination and diffusion coefficients at h_0; θ is the angle between the magnetic field and the zenith; A is the amplitude of the gravity wave at

h_0; $P_n(p, \omega, t)$ and $R_n(p, \omega, t)$ are oscillating functions of time defined in the appendix; $F_n(p)$ and $G_n(q)$ are factors that control the dependence on the scale height factor and the damping of the gravity wave, respectively, and are given in the appendix; $L_n^{-1/2}$ is a Laguerre polynomial of half-integral order.

We see at once from (2) that if we ignore the contribution from the infinite sum on the right-hand side, we have a Chapman function multiplied by an exponential damping together with oscillations. This is as would be expected, since for the nighttime situation considered here, production can be excluded. (We may easily include production if it is required.)

The infinite sum on the right-hand side of (2) reduces to a few terms for a gas that is either perfectly mixed or is in diffusive equilibrium and for a constant velocity amplitude (independent of height) for the gravity wave ($q = 0$) (see appendix).

In the case when there is no gravity wave ($A = 0$) or when we are at the geomagnetic pole ($\theta = 0$) or equator ($\theta = \pi/2$), $B(q) = 0$ (see appendix), and (1) reduces to the expression for an undisturbed ionosphere [*Moschandreas and Tuan*, 1974]. The decay constant $\lambda_0(p)$ describes the general F layer decay in the absence of disturbance and was originally derived through perturbation theory [*Tuan*, 1968]. In general, $\lambda_0(p)$ increases when the gases are in diffusive equilibrium ($p = 2$).

The expression given by (2) provides the correct phase behavior as well as the F_2 peak motion, as is seen from a comparison with *Thome*'s [1968] observations [*Porter and Tuan*, 1974]. By inserting the above expression for $N_e(z, t)$ in (1) we obtain the luminosity profiles for the 6300-Å O I shown in Figure 1. Here we have adopted ionospheric and gravity wave parameters similar to those used to fit *Thome*'s [1968] observations except for the scale height factor for which we have now used $p = 1.75$ in order to bring the gases closer to diffusive equilibrium as one might expect for the F region. We shall assume that $A_{1D} = 1/110\ s^{-1}$ [*Chamberlain*, 1961], $S_1 = 5 \times 10^{-11}\ cm^3\ s^{-1}$ [*Peterson and Van Zandt*, 1969], and $S_2 = 1.7 \times 10^{-9}\ cm^3\ s^{-1}$ [*Seaton*, 1956].

In Figure 1 a comparison of the electron density profile during the passage of a gravity wave with the movements of the luminosity profile seems to show that (1) in general, the luminosity profile peaks about one scale height below the F_2 peak as it does in the undisturbed time-dependent theory [*Tuan*, 1969], (2) 1 hour later, as the F_2 peak moves down by 20 km losing all the time in electron density magnitude, the luminosity peak remains relatively unchanged in position but greatly enhanced in intensity (Figure 1, center), and (3) another hour later, as the F_2 peak moves up again, the intensity of peak luminosity again drops, there being little rise in position (Figure 1, right). If we had plotted the luminosity profiles without quenching, the peak luminosity would have moved a considerable distance (about 15 km) downward when the F_2 peak came down. This conclusion implies that the luminosity peak is essentially kept up by the rapid increase in quenching at lower altitudes. The reason that it would only move 15 km rather than the 20 km for the F_2 peak is the rapid drop in available O_2^+ for the production of ^1D states at higher altitudes, which tends to keep the luminosity peak down. Thus the combined effect of both is to keep the luminosity peak from having larger oscillations in the vertical direction.

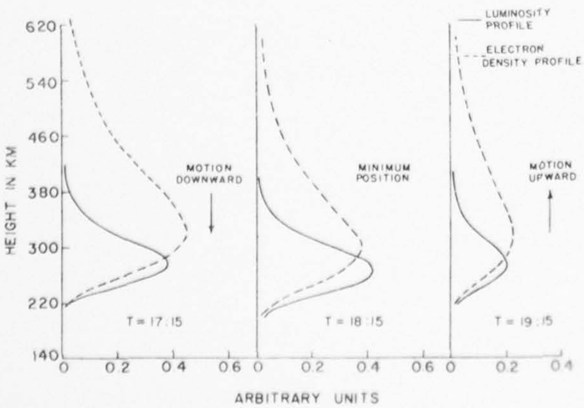

Fig. 1. Comparison of the motion of the electron density profile with 6300-Å luminosity profile under the influence of a gravity wave.

OSCILLATIONS IN TOTAL ELECTRON CONTENT AND 6300-Å O I

To obtain the total zenith airglow intensity for the 6300-Å O I, we integrate (1) over height. The results are plotted against local time in Figure 2. We have also plotted the total columnar electron content as well as the negative time derivative of this total electron content. The parameters used here are the same as those adopted for Figure 1. One general feature is that all three curves decay with time, an expected result, since no ion pair production mechanisms have been included. The most significant feature is that although there is considerable oscillation with time in the airglow intensity, the total electron content shows relatively little fluctuation. The lack of fluctuation in the total electron content is readily understandable. We are first of all dealing with a nighttime situation, when we may neglect photoproduction. The fluctuations arise when the ions are moved alternately into rarified atmosphere, where the recombination is less, and denser atmosphere, where the recombination is greater. The fluctuation is further inhibited by phase cancellation between the motion of ions above and below the F_2 peak. Since we are dealing with a horizontally ducted neutral wave with no vertical phase progression, the phase variation of ions depends almost entirely on the shape of the F layer. Figure 6 shows how the phase of the ion motion above the vicinity of the F_2 peak is in general opposite to that below the peak. We have here an example of possible phase cancellation even when the gravity wave does not possess phase variation along the line of sight. *Georges and Hooke* [1970] have shown how phase cancellation may occur from phase variations in the gravity wave itself.

The columnar airglow intensity, on the other hand, is proportional to the ion pair recombination if we neglect quenching. By integrating the continuity equation for electrons over z and dropping the production term we obtain the result that the columnar recombination rate is equal to the negative of the rate of change of columnar electron content, since the divergence term in the continuity equation drops out from Gauss theorem. Thus if the quenching were neglected, we would expect the airglow intensity to be directly proportional to the negative of the rate of change of total electron density. This result is shown very clearly in Figure 2, in which the airglow fluctuations closely follow the negative of the rate of change of total electron content. Both curves are opposite in phase to the total electron content, as might be expected. If the quenching terms are very large, then a phase difference will be introduced between the airglow fluctuations and the rate of change of total electron content. In general, there are two possible ways to introduce this phase difference,

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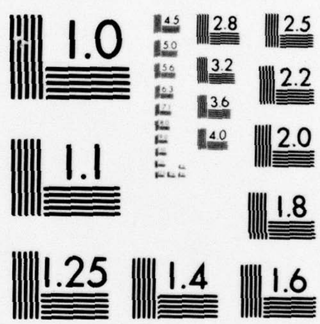

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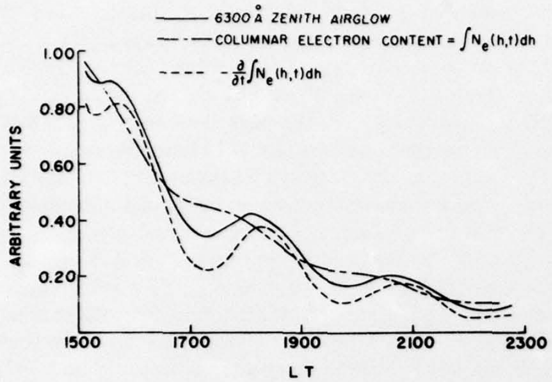

Fig. 2. Time variation of the 6300-Å zenith airglow compared with the total columnar electron content and the time derivative of the total electron content.

quenching as we have just mentioned or to introduce some production process. This point will be discussed further later.

5200-Å NITROGEN LINE

The 5200-Å N I is produced in the *F* region by dissociative recombination in a manner similar to the production of the 6300-Å line. Analysis of the effects of a gravity wave on this line is of special interest because of the very long lifetime of the nitrogen $^2D_{5/2}$ states. We can no longer use the quasi-equilibrium theories, especially at higher altitudes, where quenching can no longer compensate for the very slow radiative loss. If we neglect diffusion [*Hernandez and Turtle*, 1969], the continuity equation for the atomic nitrogen density in the ^2D state N(^2D), is given by

$$\frac{\partial N(^2D)}{\partial t} = P - (A_{^2D} + K_1[O_2] + K_2N_e)N(^2D) \qquad (3)$$

In (3), *P* is the production rate for N(^2D), K_1 and K_2 are the quenching coefficients for O_2 and electrons, respectively, $A_{^2D}$ is the Einstein coefficient for N(^2D). If we go down to low enough altitudes, where the quenching is sufficiently strong to balance the production rate, we may neglect the time derivative term, and the theory reduces to a quasi-equilibrium theory exactly analogous to that of *Peterson et al.* [1966]. For the production rate we again assume that the production of N(^2D) through dissociative recombination of NO^+ is balanced by charge transfer between O^+ and N_2. In this way, the production rate of N(^2D) states varies with height as the product of neutral N_2 concentration and N_e.

The loss rate of N(^2D) is represented by the three terms on the right-hand side of (3), corresponding to radiative loss, O_2 quenching, and quenching due to superelastic collisions with the electrons. We have assumed that $A_{^2D} = 1/26$ h^{-1} [*Chamberlain*, 1961] and $K_2 = 8 \times 10^{-10}$ cm^3 s^{-1} [*Seaton*, 1956]. For the rate coefficient K_1 with respect to O_2 quenching we have tried both $K_1 = 1.4 \times 10^{-11}$ cm^3 s^{-1} [*Slanger et al.*, 1971] and $K_1 = 1.4 \times 10^{-12}$ cm^3 s^{-1} [*Hernandez and Turtle*, 1969]. The two values differ by an order of magnitude, and there is reason to believe that the value given by Hernandez and Turtle may be on the small side (G. Hernandez, private communication, 1973). Equation (3) may be simply integrated, subject to the initial boundary conditions. From the N(^2D) concentration we may readily calculate the columnar zenith airglow intensity for the ($^4S_{3/2}$-$^2D_{5/2,3/2}$) forbidden optical transition as a function of time. The results are shown by the solid curve in Figure 3 for the quenching coefficient of *Slanger et al.* [1971] and by the dotted curve for the coefficient used by *Hernandez and Turtle* [1969]. For both curves we have used a scale height factor of $p = 1.75$. All other gravity wave parameters are the same as those used to fit Thome's results [*Porter and Tuan*, 1974].

The three important features are that (1) the oscillations for the higher O_2 quenching rate of *Slanger et al.* [1971] are less pronounced than those produced by the lower quenching coefficient of *Hernandez and Turtle* [1969], (2) there is a phase advance with the higher quenching coefficient of between 10 and 20 min (the reasons for this will be given in a later section), and (3) the luminosity peak for the higher quenching coefficient is in general about 20 km higher than the peak for the lower coefficient.

In comparing these results with the behavior of the 6300-Å line, there are the following important differences: The electron quenching for the nitrogen line is, relatively speaking, much more important than that for the oxygen line. If we use the coefficient of *Slanger et al.* [1971], the quenching due to molecules for the nitrogen line is down by more than an order of magnitude from the quenching for oxygen, owing to the combined effect of a smaller quenching coefficient and a smaller abundance of O_2 molecules than of N_2 molecules. The difference in electron quenching is only about a factor of 2.

The relatively more important electron quenching has the effect of holding down the 5200-Å luminosity peak, since the available electrons for quenching increase until the F_2 peak, well above the luminosity peaks of both the 6300-Å and the 5200-Å line.

The net result is that although one may at first expect the 5200-Å luminosity peak to be well above the oxygen peak, owing to a much longer half life, the reduced molecular quenching at lower altitudes coupled with a relative increase in electron quenching (unimportant for the 6300-Å line) at higher altitudes would bring down the peak position for the 5200-Å line to height levels comparable to those for the 6300-Å line. This is not inconsistent with the observations of *Weill and Christophe-Glaume* [1967], who argued from geometric considerations that the 5200-Å peak is about 12 km above the 6300-Å peak if the speed of the traveling ionospheric disturbance is assumed to be the same at the peaks of both emissions. We may conclude that since both peaks occur at nearly the same height, the oscillation amplitudes should be comparable. This conclusion is borne out by comparing Figure 3 with Figure 2.

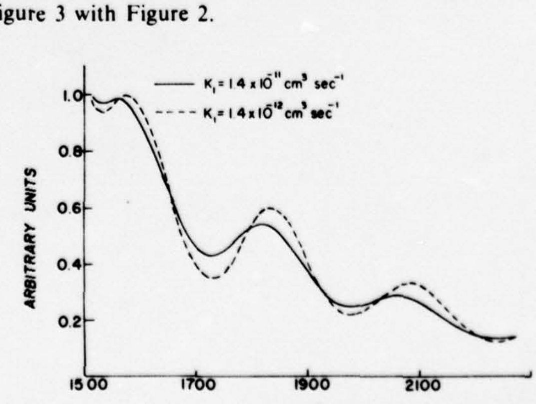

Fig. 3. Time variation of the 5200-Å nitrogen emission in a gravity wave for the O_2 quenching coefficient of *Hernandez and Turtle* [1969] (dashed curve) and *Slanger et al.* [1971] (solid curve).

COMPARISON WITH SIMULTANEOUS OBSERVATIONS OF f_0F_2, $h'F_2$, AND 6300-Å O I EMISSION

We have mentioned earlier that some published data seem likely to have exhibited the effects of gravity waves. In this section we use some of *Dachs*'s [1968] simultaneous measurement of ionospheric parameters and the 6300-Å O I emission to illustrate the theory by fitting the observations to an assumed gravity wave.

Dachs [1968] has made intensity measurements of the 6300- and 5577-Å emission lines as a function of time. These measurements together with simultaneous measurements of f_0F_2 and $h'F_2$ seem to show considerable correlation, especially between the 6300-Å line and the ionospheric parameters. In Figure 4 we have shown a comparison between our theoretical results and the experimental values for f_0F_2, $h'F_2$, and the 6300-Å airglow. For the theoretical parameters we have chosen p = 1.5, H = 65 km, the recombination coefficients β(200 km) = 0.34 h^{-1}, and the diffusion coefficient D(200 km) = 6.1 × 10^3 km^2 h^{-1}. These are somewhat different from the parameters chosen to fit Thome's observation, but considering the fact that we have completely ignored production processes (the observations are made between 1800 and 2400 LT), they are the right order of magnitude. For the gravity wave we assume the velocity amplitude A(200 km) = 35.2 m s^{-1} and the damping factor q = 0.49, which is rather close to *Hines*'s [1960] undamped model. The quenching at 200 km is assumed to be 0.24 s^{-1}, and the electron quenching is neglected. The quantum efficiency for emission from the ^1D is assumed to be 0.5.

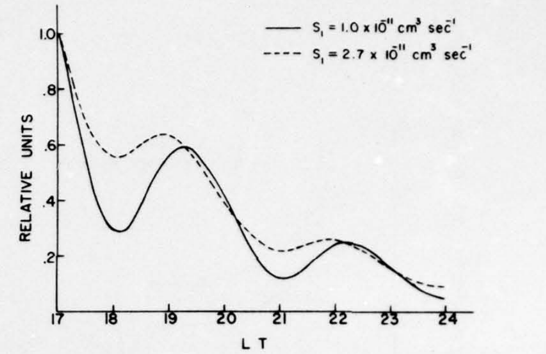

Fig. 5. Oscillations of the 6300-Å line for two nitrogen quenching coefficients, showing how the larger quenching coefficient (dashed curve) reduces oscillation and introduces a phase advance.

In view of the rather good agreement between theory and experiment for both the airglow and the ionospheric parameters one can readily conclude that even without a total electron content measurement, we have a gravity wave passing through with a period of between 2 and 3 h. The comparison between the airglow intensity and the $h'F_2$ shows all the expected phase correlations; i.e., when the airglow intensity is at a minimum, the F_2 peak is approximately at a maximum.

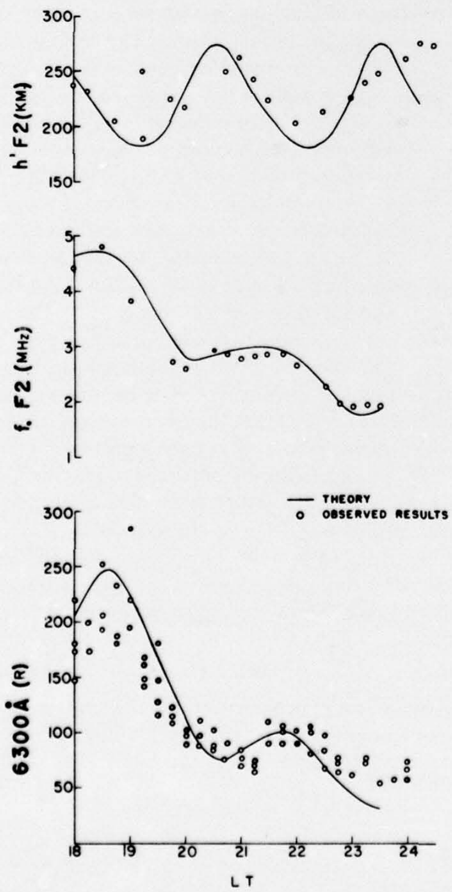

Fig. 4. Experimental observations (open circles) of Dachs showing simultaneous variations in f_0F_2, $h'F_2$, and the 6300-Å airglow compared with theoretical calculations (solid curves) based on the assumption of a gravity wave with a period of approximately 3 h.

EFFECT OF QUENCHING AND THE CRITERIA FOR DIFFERENT DISTURBANCES

As was already mentioned in a previous section, quenching can cause a phase difference in the oscillation of the airglow intensity. Figure 5 shows the 6300-Å O I for two N_2 quenching coefficients. The solid curve uses a quenching coefficient S_1 = 1.0 × 10^{-11} cm^3 s^{-1}, whereas the dashed curve uses S_1 = 2.7 × 10^{-11} cm^3 s^{-1}. All other parameters are the same as those used to fit *Dachs*'s [1968] experimental observations. The higher quenching coefficient seems to produce fewer oscillations as well as a phase advance by about half an hour. It is easy to understand why the oscillation should decrease when the quenching is increased, since a higher quenching rate would move the luminosity peak up to a point where the electron density oscillation is less. This is true so long as the peak luminosity remains below the F_2 peak [*Porter and Tuan*, 1974].

To understand why we should have a phase advance, we must consider the variation in the phase of the electron density oscillation with respect to the neutral gravity wave as a function of height. The solid curve in Figure 6 shows a plot of the theoretical phase variation with altitude, and the dashed curve shows *Thome*'s [1968] observed results. (Figure 6 is taken from *Porter and Tuan* [1974].) We see that the phase of the electron density oscillations advances monotonically with height. There is a region, however, below 260 km where an increase in the height of the luminosity will bring little phase change. Above 260 km any increase in the height of the luminosity curve would bring a rapid advance in the phase of the electron density oscillation. Even though Figure 5 deals with an integrated intensity, since the phase advance is monotonic, an upward shift of the entire luminosity profile must always produce a phase advance.

The analysis seems to suggest a means for distinguishing airglow fluctuations caused by gravity waves from those caused

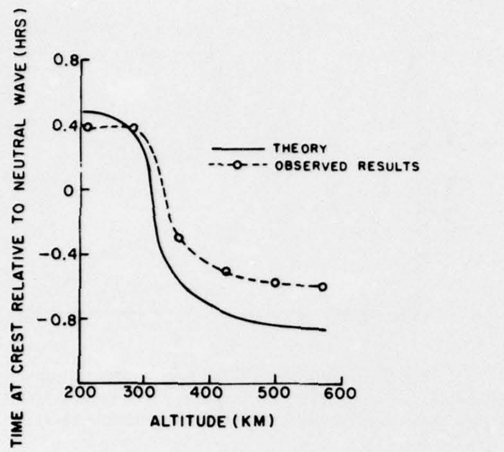

Fig. 6. Theoretical calculations of the variation with altitude in the phase of the ionospheric oscillations compared with *Thome*'s [1968] observed values.

by incident particle flux, if a simultaneous measurement of total electron content and airglow intensity can be made. If no production mechanism is present and the disturbance is caused entirely by a gravity wave, then the following observations are true:

1. There is relatively little fluctuation in the total electron content or in any experimental parameter that depends only on the total number of columnar electrons rather than, say, on how the electrons are distributed.

2. The rate of change of total electron content should remain negative. Thus for nighttime conditions and a gravity wavelength that is sufficiently long (of the order of 1500 km, say) in comparison with the thickness of the F layer that we may neglect horizontal transport arising from concentration gradients in the horizontal direction, the total columnar electron content should decrease monotonically with time. This result is easily understandable, since the only reason for the little fluctuations that we do get in the total electron content is the variation in the recombination coefficients as the ionosphere moves up and down.

3. There is always a phase advance in the airglow fluctuations with respect to the rate of change of total electron content. The magnitude of the advance is in general determined by the quenching.

We should remark here that if a horizontal time-varying electrostatic field exists in the F region causing an $\mathbf{E} \times \mathbf{B}$ drift in the ambient electrons only, without any horizontal transport, then all three of the above properties in the behavior of the airglow and columnar electron density should also hold. They would no longer be true, however, if somehow additional electrons were introduced either through transport or some sort of production mechanism. For instance, if the disturbance is caused by an incident particle flux that produces additional ion-electron pairs, then the rate of change of total electron content can go positive, since the total columnar electron content can no longer vary monotonically with time. In general, the fluctuation can also be far more drastic [*Moschandreas and Tuan*, 1974].

As an example we consider the data of *Brown and Steiger* [1972], who have simultaneously measured the 6300-Å airglow and the total electron content as a function of time at Hawaii. They found an oscillation in the airglow that correlates with oscillations in the total electron content. The negative time derivative of columnar electron content trails the airglow oscillation by a time delay of about 25 min. We have at first some reason to feel that this particular disturbance could be caused by a wave, since (1) the disturbance appears to be periodic with a period of about 3½ h, and (2) the airglow fluctuation shows a phase advance of about 20–30 min with respect to the rate of change of total electron content. However, it is rather unlikely that the disturbance is a wave, since the period is of the order of 3½ h and the disturbance takes place around the middle of the night, when we can forget about photoionization. The disturbance also does not vary monotonically with time, and it shows large oscillation amplitudes. We may add that the first condition mentioned above can be produced by any periodic disturbance, and the second condition can be produced by either a gravity wave or a vertical drift without any introduction of additional charged particles. There is of course a possibility that we may simultaneously have an electromagnetic drift together with an introduction of additional electrons. In fact, such an event would be quite consistent with both the experimental observations and our criteria.

CONCLUSION

We have examined the effect of a gravity wave in the F region on the electron density and on the emission intensity of two forbidden transitions whose mechanisms involve recombination with electrons. Because of such effects as quenching and the lack of available O_2^+ and NO^+ molecules at high altitudes, the vertical movement of the luminosity profiles in response to the passage of the gravity wave is much smaller than that of the electron density profile. The amplitude of oscillation of the airglow, on the other hand, is greater than that of the total electron density, which varies monotonically with time. Owing to the presence of a magnetic field as well as the shape of the electron density profile, phase differences in behavior occur between different heights. This effect, combined with the height-differentiating effect of quenching, produces phase differences also between airglow and ionospheric parameters. A combination of airglow and ionospheric measurments can be used to differentiate between gravity waves and the effects of incident particle flux.

The theory has been tested by assuming a gravity wave effect for some published data on simultaneous measurements of airglow and ionospheric parameters. The fit was satisfactory.

This study represents the first theoretical treatment, as far as we are aware, of the effects of gravity waves on airglow. We have tried here to establish the basic characteristics to be expected for the luminosity and electron density profiles under the influence of a gravity wave and to show how these could be used in some cases to provide information on processes. Expansion of the techniques should be useful as a tool for the study of both gravity waves and upper-atmosphere parameters and processes.

APPENDIX

A good analytical approximation for the decay constant λ_0 expressed as a function of the scale height factor p is given by

$$\lambda_0(p) = \frac{(B_0 D_0)^{1/2}}{2H} \cdot \left\{1 + \frac{1}{\pi^{1/2}}\left[\alpha^{1-p}\Gamma(p + \tfrac{1}{2}) - \Gamma(1 + \tfrac{1}{2})\right]\right\} \quad \text{(A1)}$$

A derivation for this is given by *Tuan* [1968]. Another totally different derivation has been given by *Porter and Tuan* [1974]. In this expression, λ_0 decreases monotonically with increasing p, where $1 \leq P \leq 2$. Thus the decay of the F layer is in

general faster when the gases are perfectly mixed ($P = 1$).

The expression $B(q)$ in (2) is given by

$$B(q) = \frac{A\alpha^q \sin\theta \cos\theta\Gamma[(3/2) - q]}{2\pi^{1/2}\omega H} \tag{A2}$$

As was already mentioned, q varies between 0 and ½. If q is set to zero, the velocity amplitude does not vary with height. Otherwise, it increases exponentially. We see that $B(q)$ vanishes at the magnetic pole and equator, where we do not expect a horizontally ducted gravity wave to produce oscillations in airglow.

The functions $P_n(p, \omega, t)$ and $R_n(p, \omega, t)$ are defined [*Porter and Tuan*, 1974] by

$$P_n = c(q, t_0, \bar{t}_0) \int_{t_0}^{t} \exp\left[\frac{\omega_n(t - t')}{k}\right] \cdot \exp -[\lambda_0(t' - t_0) - B(q) \sin \omega(t' - \bar{t}_0)]\, dt' \tag{A3}$$

$$R_n = c(q, t_0, \bar{t}_0) \int_{t_0}^{t} \exp\left[\frac{\omega_n(t - t')}{k}\right] \cdot \exp -[\lambda_0(t' - t_0) - B(q) \sin \omega(t' - \bar{t}_0)] \cdot \cos \omega(t' - \bar{t}_0)\, dt' \tag{A4}$$

where

$$\omega_n = -(n + \tfrac{1}{4})$$

$$k = H/2(B_0 D_0)^{1/2}$$

Although both P_n and R_n are oscillating functions of time with frequency ω, R_n may change sign, but P_n remains always positive. It is easy to see that as $\omega \to 0$, both P_n and R_n become monotonic functions of time, and all oscillations vanish from the expression given by (2).

The functions $F_n(p)$ and $G_n(q)$ are given by

$$F_n(p) = \frac{\alpha^{1-p}\Gamma(n - p)\Gamma(n + \frac{1}{2})}{\Gamma(-p)} - \frac{\Gamma(n - 1)\Gamma(1 + \frac{1}{2})}{\Gamma(-1)} \tag{A5}$$

$$G_n(q) = \frac{\Gamma(q + n - 1)\Gamma[(3/2) - q]}{\Gamma(q)} \left[\frac{(1 - q)}{2} - n\right] \tag{A6}$$

The derivation of these functions is given by *Tuan* [1968] and *Porter and Tuan* [1974]. They have the following important properties:

$$F_n(p = 1) = 0 \qquad \forall n$$

$$F_n(p = 2) = 0 \qquad \forall n \geq 3$$

$$G_n(q = 0) = 0 \qquad \forall n \geq 2$$

Hence the infinite sum on the right-hand side of (2) will reduce to only a few terms for a gas that is either perfectly mixed or in diffusive equilibrium and for a constant velocity amplitude ($q = 0$).

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